# The Lookahead Limitation: Why Multi-Operand Addition is Hard for LLMs

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## Abstract

Autoregressive large language models (LLMs) exhibit impressive performance across various tasks but struggle with simple arithmetic, such as additions of two or more operands. We show that this struggle arises from LLMs' use of a **simple one-digit lookahead heuristic**, which forms an upper bound for LLM performance and works fairly well (but not perfect) for two-operand addition but fails in multi-operand cases, where the carry-over logic is more complex. Our probing experiments and digit-wise accuracy evaluation show that the evaluated LLMs fail precisely where a one-digit lookahead is insufficient to account for cascading carries. We further validate our findings via a targeted circuit intervention, revealing a specialized pathway responsible for direct carry detection from the next digit position. We analyze the impact of tokenization strategies on arithmetic performance and show that all investigated models, regardless of tokenization and size, are inherently limited in the addition of multiple operands due to their reliance on a one-digit lookahead heuristic. Our findings reveal fundamental limitations that prevent LLMs from generalizing to more complex numerical reasoning.

## 1 Introduction

Large language models (LLMs) demonstrate remarkable performance across a wide range of tasks (Bai et al., 2023; Team et al., 2024; Guo et al., 2025), yet struggle with simple arithmetic tasks, such as the addition of multiple or large numbers - sometimes even after finetuning (McLeish et al., 2024; Shen et al., 2023; Zhou et al., 2023, 2024a).

The difficulty LLMs face in arithmetic tasks stems from the mismatch between the left-to-right nature of autoregressive language modeling and the right-to-left structure of standard arithmetic algorithms. Conventional addition methods process numbers digit by digit from right to left, propagating carries, while LLMs generate numbers sequentially from left to right without explicit intermediate calculations. This raises the question: What strategy do LLMs use to handle this misalignment in addition?

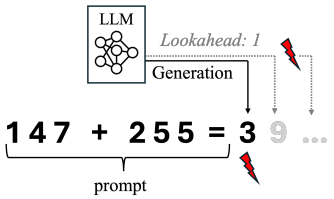



Figure 1: An addition of two three-digit operands. LLMs rely on a one-digit lookahead when performing addition. If a relevant carry emerges at a later stage in prediction, they fail to account for it, leading to errors in earlier generated result digits.

In this work, we show that pretrained LLMs rely on a simple heuristic that enables high accuracy in two-operand addition (e.g., $147 + 291 = 438$) but produces systematic and predictable error patterns error patterns. This heuristic attempts to bridge the gap between the left-to-right generation and the resulting need to 'look ahead' to account for carries from less significant digits. Rather than performing an full lookahead to anticipate cascading carries, **LLMs rely on a single-digit lookahead heuristic** to estimate carries. We demonstrate that this lookahead depth is limited to one digit across all pretrained models investigated. While this strategy works reasonably well for two-operand addition, it deteriorates substantially with multiple operands where cascading carries are more frequent, due to relevant digit combinatorics (henceforth generalized as *multi-operand addition* for any number of operands $> 2$).

Figure 1 illustrates a characteristic failure of this heuristic: because the model can only look one digit ahead, it bases its prediction on the middle digits (e.g., $4 + 5 = 9$), incorrectly infers that no carry will occur, and consequently predicts the first

result digit as 3, failing to propagate the carry from the units position.

To test whether the heuristic accurately describes the strategy used by LLMs to solve addition from left to right, we present results from four state-of-the-art LLMs with different tokenization strategies (single digit and multiple digit) for numerical outputs. Using carefully curated datasets, digit-wise accuracy metrics, and probing techniques, we show that errors emerge precisely in cases where a one-digit lookahead is insufficient to handle cascading carries.

Finally, we identify a circuit that contributes information about direct carries from the next digit position to the current generation step, and causally verify its role through targeted interventions, confirming the presence of single-digit lookahead.

Our findings show that all investigated LLMs are constrained by a limited lookahead heuristic, independent of tokenization strategy and model size, revealing a fundamental algorithmic limitation in their arithmetic reasoning.

Our contributions are as follows:

- **Evaluation of Addition Capabilities**: We show that LLMs fail on multi-operand addition (Section 2) and then systematically evaluate the capabilities of LLMs on two-operand addition tasks via probing (Section 3).
- **Heuristic Discovery**: Inspired by results of the evaluation, we formalize left-to-right addition in LLMs for multi-operand addition with a simple heuristic that uses a lookahead of one for left-to-right addition (**H1**, Section 4).
- **Empirical Validation**: We demonstrate that **H1** is fragile in multi-operand addition and explain the performance decline as a function of the increasing number of operands in large comprehensive addition experiments. We find that model performance aligns *precisely* with the predicted limitations of **H1** (Sections 5 and 6). We find that **H1** holds independently of tokenization strategies (Section 7).
- **Carry Circuit Discovery and Causal Verification**: We identify a circuit that contributes information about direct carries from the next digit position to the current generation step and confirm its causal role through targeted interventions, thereby providing mechanistic evidence for single-digit lookahead.

## 2 LLMs Struggle with Multi-Operand Addition

We now define the data and models used in this work and demonstrate that LLMs fail on multi-operand addition via prediction accuracy.

### 2.1 Models and Data

**Models.** We compare Mistral-7B (Jiang et al., 2023), Gemma-2-2B and Gemma-2-9B (Team et al., 2024) and Meta-Llama-3-8B (Grattafiori et al., 2024; AI@Meta, 2024) as they employ different tokenization strategies for numerical outputs: While Mistral and Gemma-2 exclusively employ a single-digit tokenization strategy for their numeric input and generated output (e.g., input = ['1', '4', '7', '+', '2', '5', '5', '='], output = ['4', '0', '2']), Llama-3 employs a multi-digit numeric tokenization strategy (e.g., input = [' 147', ' +', ' 255', ' ='], output = [' 402']), typically favoring numeric tokens of length 3.

**Data.** For all experiments in this paper, we compile a range of datasets containing simple arithmetic task prompts of the form *147 + 255 =* . We create a dataset for each addition task ranging from 2-operand to 11-operand addition, where each operand is a triple-digit number between 100 and 899. Each of the 10 datasets contains 5,000 unique arithmetic problems, both in a zero-shot and one-shot setting. In the zero-shot setting, an example for a 2-operand addition prompt is "147 + 255 = ". An example for a 4-operand addition prompt is "251 + 613 + 392 + 137 = ". Our one-shot prompt template follows the scheme *q1 r1; q2* , e.g. "359 + 276 = 635; 147 + 255 = ", where *q1* is a sample query from the same dataset and *r1* is the correct result of the addition task in *q1*. *q2* is the query containing the addition task to be solved.

In the remainder of the paper, we use $s_n$ (with $n \geq 0$) to denote the result digit generated at digit position $10^n$. For example, in "147 + 255 =", with expected output 402, $s_2 = 4$, $s_1 = 0$, and $s_0 = 2$.

### 2.2 LLM Accuracy on Addition Tasks

Figure 2 illustrates the significant decline in performance of Mistral-7B, Gemma-2-2B, Gemma-2-9B and Llama-3-8B in multi-operand addition as the number of operands increases. This drastic decrease highlights the inability of these models to generalize effectively to addition tasks involving a higher number of operands, despite their strong overall capabilities.

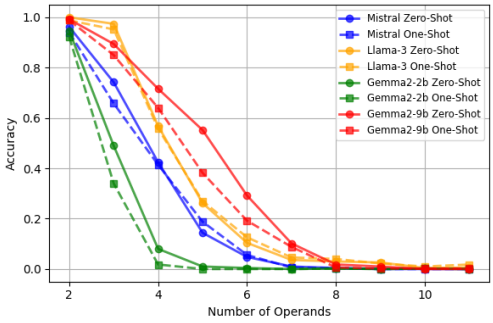



Figure 2: Accuracy of Mistral, Gemma-2 variants and Llama-3 on multi-operand addition of triple-digit numbers, in a zero- and one-shot setting.

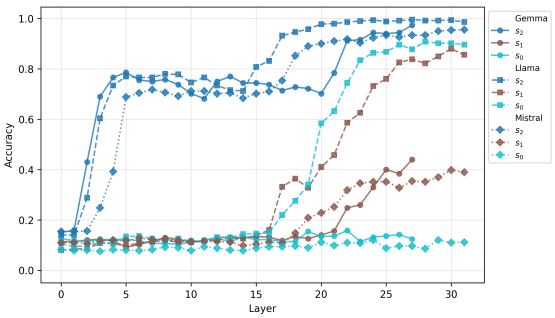



Figure 3: Probing accuracy of individual result digits as predicted by the hidden states of Mistral, Gemma-2-9B and Llama-3. For two-operand, zero-shot addition prompts.

## 3 Probing LLMs on Digits in Two-Operand Addition Tasks

Solving arithmetic tasks presents a fundamental challenge for LLMs, as they generate text from left to right, while addition requires a right-to-left process due to carry propagation from the least significant to the most significant digit. For instance, predicting the first result digit $s_2 = 4$ in "147 + 255 = " requires the model to anticipate that a carry originating from $s_0$ cascades through $s_1$ to $s_2$. Robust left-to-right addition thus requires a lookahead spanning all result digits, raising the question: Do LLMs internally represent future result digits when predicting $s_2$ - and if so, how far can they "look into the future"?

To answer this question, we probe whether models accurately encode future result digits $s_1$ or $s_0$ while generating $s_2$. Building on Levy and Geva (2024), who show that, irrespective of a model's numeric tokenization strategy, LLMs internally represent numbers digit-by-digit in base 10, we analyze digit-wise probing accuracy on the two-operand addition dataset described in Section 2.1.

### 3.1 Methodology and Experiments

**Data.** We split the two-operand addition dataset (see Section 2.1) into train (n=4500) and test (n=500) for the probing experiments. The two-operand addition dataset is designed such that correct results for the addition tasks are triple-digit numbers between 200 and 999. We use the zero-shot prompt setting for the probing experiment.

**Probing Setup.** Our goal is to determine which result digits are available at the prediction step of $s_2$. We thus train probes to predict the result digits $s_2$, $s_1$, and $s_0$ from hidden states of the model during the prediction step of $s_2$.

Specifically, we train one-layer linear probes to predict individual digit values of the results from the hidden state of the last token at each model layer. Probes are trained on the train split of the two-operand addition dataset and evaluated on the test split. We train separate probes to predict individual result digits $s_2$, $s_1$, and $s_0$, for all models at all layers.[1]

### 3.2 Results

The probing accuracy of individual result digits is shown in Figure 3. Gemma-2 and Mistral with their digit-wise tokenization internally represent only $s_2$ with high accuracy. In contrast, there is a high probing accuracy across *all* result digits in Llama-3. This is due to the fact that Llama-3 tokenizes numbers into 3-digit numeric tokens: It is forced by its tokenization to generate all result digits ($s_2$, $s_1$, and $s_0$) in one step as a single token.

The single-digit tokenization models Mistral and Gemma-2 exhibit a low probing accuracy on $s_0$ ($< 0.24$) in all layers. Recall that $s_0$ is probed from the models' hidden states while they autoregressively generate $s_2$. We interpret the lack of internal representation of $s_0$ as evidence that these models disregard the potential influence of $s_0$ (including any cascading carry) when generating $s_2$.

In line with this, Gemma-2 and Mistral show notably higher probing accuracy on $s_1$ compared to $s_0$, when probing from the models' hidden states as they generate $s_2$. We thus conjecture that the single-digit-token models seem to recognize the potential

[1]We choose a low temperature of 0.1 during model inference to ensure deterministic and consistent outputs, reducing randomness in token generation and improving the reliability of numerical calculations.

influence of the carry resulting from the sum of the $10^1$ operand digits. Simply put, generating the digit at $10^2$ might employ a lookahead of one digit to the $10^1$ intermediate result. Based on this observation, we formulate a hypothesis for a heuristic used by LLMs:

**H1: LLMs employ a look ahead of one digit to generate the current digit of an addition task.**

**H1** would explain why LLMs cannot effectively represent each necessary digit of the result during generation, making it difficult to anticipate later carry values correctly. We first formalize **H1**, which explains the patterns observed in Figure 3, in the next Section, and then verify the fit of **H1** with empirical addition outcomes generated by the models in Sections 5, 6, and 7.

## 4 The Carry Heuristic of LLMs

Since LLMs generate numbers from left to right, they must anticipate whether a carry from later digits (with lower bases further on in the result) will impact the current digit they are generating. In this section, we evaluate the maximum accuracy LLMs can achieve in addition tasks, assuming they rely on **H1**, given the limited lookahead of one digit.

### 4.1 Formalization of Left-to-Right Addition

We first formalize a recursive algorithm for solving addition of $k$ operands – where each operand is a base 10 integer – in a left-to-right manner.
**We define:**

- $k$: Number of operands.
- $n_1, n_2, \ldots, n_k$: Operands, each represented as digit sequences in base 10, with $0 \leq i < d$, where $d$ is the number of digits in the operands: $n_j = [n_{j,d-1}, \ldots, n_{j,0}]$, $n_{j,i} \in \{0, \ldots, 9\}$
- $S$: The result of the addition. $S = [s_d, s_{d-1}, \ldots s_0]$, where $s_d = c_d$, i.e., the final carry.

We recursively define the calculation of individual result digits:

- **Total Sum at Digit Position $i$:**

$$t_i = \sum_{j=1}^{k} n_{j,i}$$

$$T_i = t_i + c_i$$

where $t_i$ is the digit sum at the current position, $c_i$ the carry from the previous digit position, and $k$ the number of operands. Base case: $c_0 = 0$, no carry at the least significant digit.

- **Result Digit at Position $i$:**

$$s_i = T_i \mod 10$$

- **Carry to the Next Digit Position:**

$$c_{i+1} = \left\lfloor \frac{T_i}{10} \right\rfloor$$

A worked example is provided in Appendix A.

### 4.2 A Naive Heuristic for Solving Addition Left-to-Right

Due to the recursive nature of left-to-right addition, a lookahead of $i-1$ digits is needed to determine any result digit $s_i$. There is however a simple, non-recursive heuristic for the estimation of $s_i$ with only a one-digit lookahead, to the digit sum of the next position, i.e. only considering $t_{i-1}$.

We define $c_{min}$ and $c_{max}$ to be the minimal and maximal possible value for a carry, where trivially for all cases, $c_{min} = 0$, and

$$c_{max}(k) = \left\lfloor \frac{\sum_{j=1}^{k} 9}{10} \right\rfloor$$

in base 10 and for $k$ operands. We then define the carry heuristic $c_i^h$ as follows:

$$c_i^h \in \{ \left\lfloor \frac{t_{i-1} + c_{min}}{10} \right\rfloor, \left\lfloor \frac{t_{i-1} + c_{max}}{10} \right\rfloor \}$$

Where $c_i^h$ is chosen uniformly at random. We then accordingly define the predicted total sum at digit position i

$$T_i^h = t_i + c_i^h$$

and the predicted result digit

$$s_i^h = T_i^h \mod 10$$

**Examples.** We show two examples of two-operand addition, one in which **H1** is successful, and one in which it fails. For $k = 2$, i.e., in two-operand addition:

$$c_{max}(2) = \left\lfloor \frac{\sum_{j=1}^{2} 9}{10} \right\rfloor = 1$$

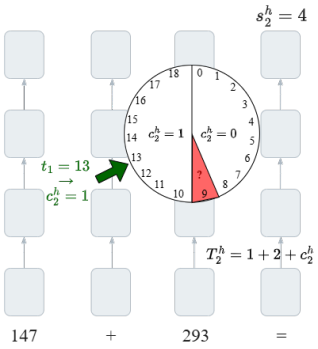



Figure 4: Two-operand addition in which **H1** is successful.

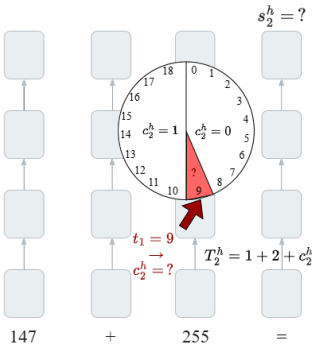



Figure 5: Two-operand addition in which **H1** fails.

**147 + 293.** See Figure 4. We need $T_2^h$ and thus $c_2^h$ to generate the first result digit $s_2^h$.

$$c_2^h \in \{\left\lfloor \frac{4+9+c_{min}}{10} \right\rfloor, \left\lfloor \frac{4+9+c_{max}}{10} \right\rfloor\}$$

$$= \{\left\lfloor \frac{13}{10} \right\rfloor, \left\lfloor \frac{14}{10} \right\rfloor\} = \{1, 1\}$$

therefore $c_2^h = 1$, $T_2^h = 4$, and $s_2^h = 4$. **H1** succeeds in predicting the first digit $s_2$ for **147 + 293**.

**147 + 255.** See Figure 5.

$$c_2^h \in \{\left\lfloor \frac{4+5+c_{min}}{10} \right\rfloor, \left\lfloor \frac{4+5+c_{max}}{10} \right\rfloor\}$$

$$= \{\left\lfloor \frac{9}{10} \right\rfloor, \left\lfloor \frac{10}{10} \right\rfloor\} = \{0, 1\}$$

therefore $c_2^h$ is chosen uniformly at random between 0 and 1. The heuristic fails in predicting the first digit $s_2$ for **147 + 255** with a 50% chance.

## 5 H1 Predicts Difficulties of LLMs in Two-Operand Addition

In this section we show that single-digit token LLMs struggle exactly in those cases in which the heuristic **H1** is insufficient.

### 5.1 Predicted Accuracy

For two-operand addition, there are 19 possible values for each $t_i$ (ranging from 0 to 18, because this is the range of sums between two digits). In 18 out of these 19 cases, **H1** reliably determines the correct carry value. Only if $t_i = 9$, **H1** must randomly choose between two possible carry values, thus failing with a 50% chance. This results in an overall predicted accuracy of

$$\frac{18 \times 1.0 + 1 \times 0.5}{19} = 0.974$$

for the first result digit $s_2$ in two-operand addition: **H1** achieves 97.4% accuracy in correctly predicting the first result digit $s_2$. This corresponds to the high accuracies that Gemma-2 and Mistral reach in generating $s_2$ during zero-shot and one-shot inference (Gemma-2-2B: 0-shot: 92.20%, 1-shot: 94.60%; Gemma-2-9B: 0-shot: 98.80%, 1-shot: 99.20%; Mistral: 0-shot: 94.00%, 1-shot: 96.20%).

### 5.2 Finegrained Analysis

We further investigate whether it is true that especially cases with $t_i = 9$ are challenging for LLMs.

**Data.** To this end, we evaluate prediction accuracy across five distinct newly introduced datasets, each containing 100 queries with distinct carry scenarios. The datasets follow the zero-shot template described in Section 2.1 and are designed to exhaustively capture all cases of carries affecting $s_2$ in two-operand addition of triple-digit numbers.

- **Dataset 1 (DS1): No carry.** The addition does not produce any carry (e.g., $231 + 124 = 355$).[2].
- **Dataset 2 (DS2): Carry in position $10^0$, no cascading.** A carry is generated in the $10^0$ ($s_0$) digit but does not cascade to the $10^2$ ($s_2$) digit (e.g., $236 + 125 = 361$).
- **Dataset 3 (DS3): Cascading carry from $10^0$ to $10^2$.** A carry originates in the $10^0$ ($s_0$) digit and cascades to the $10^2$ ($s_2$) digit (e.g., $246 + 155 = 401$).
- **Dataset 4 (DS4): Direct carry in position $10^1$.** A carry is generated in the $10^1$ ($s_1$) digit and directly affects the $10^2$ ($s_2$) digit (e.g., $252 + 163 = 415$).
- **Dataset 5 (DS5): No carry, but position $10^1$ digits sum to 9.** There is no carry in any digit,

[2]We employ the additional constraint that the sum of the $10^1$ operand digits $\neq 9$, i.e., ($s_1 \neq 9$)

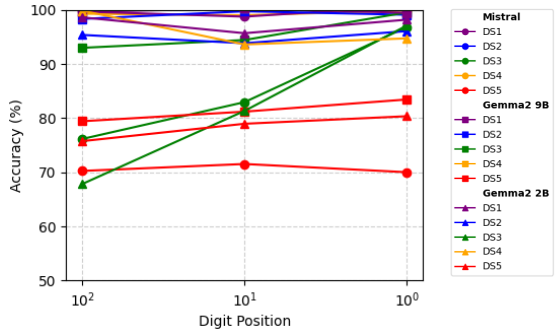



Figure 6: Per-digit generation accuracy of Mistral and Gemma-2 variants on datasets DS1-DS5.

but the sum of the $10^1$ operand digits is 9, i.e., $(s_1 = 9)$ (e.g., $256 + 142 = 398$).

DS1 to DS5 can be neatly categorized according to whether the heuristic can accurately predict $s_2$:

- DS1 and 2: $t_1 = \sum_{j=1}^{2} n_{j,1} < 9 \rightarrow c_2^h = 0$
- DS4: $t_1 = \sum_{j=1}^{2} n_{j,i} > 9 \rightarrow c_2^h = 1$
- DS3 and 5: $t_1 = \sum_{j=1}^{2} n_{j,1} = 9 \rightarrow c_2^h = ?$

**Results.** Figure 6 shows that LLMs struggle with DS3 and DS5, which are precisely the cases where **H1** predicts issues[3]. As **H1** suggests, predicting the first result digit $s_2$ at position $10^2$ is particularly error-prone in these scenarios [4]. The difficult datasets are the ones where a lookahead of one digit position does not suffice to determine the value of the carry needed to generate $s_2$. Simply put: Overall, addition results tend do be predicted correctly by LLMs, if and only if a lookahead of one digit is sufficient to determine the value of the carry bit affecting $s_2$. Prediction is often incorrect if a lookahead of two or more digits is needed to determine the value of the carry bit affecting $s_2$.

In cases where a lookahead of one digit is enough to accurately determine the value of $s_2$ (DS1, DS2, DS4), the models succeed. However, when a lookahead of one digit is insufficient to determine the value of $s_2$ (DS3 and DS5), the model struggles with predicting $s_2$ correctly. Table 1 in Appendix B provides the generation accuracy of $s_2$ for Gemma-2 and Mistral, in addition to the plot.

[3]Gemma-2-9B performs better on DS3 than expected, which suggests that the model tends to generate the carry bit when unsure.

[4]Qualitative analysis shows why digits $s_1$ in DS3, and $s_1$ and $s_0$ in DS5, are also often wrong. In DS3 (e.g., $246 + 155 = 401$), an early $s_2$ error (e.g., generating 3) leads the model to minimize the numerical difference to the correct result, producing 391 instead of 301. In DS5 (e.g., $256 + 142 = 398$), an initial error in $s_2$ (e.g.,generating 4) prompts the model to round the next hundred, giving 400 and causing errors in both $s_1$ and $s_0$.

## 6 H1 Predicts the Deterioration of Accuracy in Multi-Operand Addition

As shown in the last section, **H1** is a good approximator for LLM behaviour on two-operand addition: In the majority of cases, a lookahead of one digit is sufficient to accurately determine the value of the carry bit affecting $s_2$. With a look-ahead of one digit, **H1** predicts a failure of the generation of $s_2$, if and only if the value of $s_1$ does not suffice to determine the value of the carry bit. In two-operand addition in base 10, this is the case if and only if $t_1 = 9$. We now show that **H1** can also account for model performance on *multi*-operand addition.

### 6.1 Multi-Operand Performance Predicted by H1

The possible value of a carry increases with increasing numbers of operands. For instance in 4-operand addition ($k = 4$) the maximal value of a carry is 3:

$$c_{max}(4) = \left\lfloor \frac{\sum_{j=1}^{4} 9}{10} \right\rfloor = 3$$

Therefore the carry heuristic $c_i^h$ is unreliable in 4-operand addition whenever $t_{i-1} = \sum_{j=1}^{k} n_{j,i-1} \in \{7, 8, 9, 17, 18, 19, 27, 28, 29\}$.

Put simply, because the value of the carry can be larger for more operands, **the proportion of values of $s_1$ for which the heuristic is insufficient (with its lookahead of one) increases with an increasing number of operands**.

Consider an example in which the heuristic fails in 4-operand addition for clarification (see Figure 10 in Appendix C):
**186 + 261 + 198 + 256.**

$$t_1 = 8 + 6 + 9 + 5 = 28$$

$$c_2^h \in \{ \left\lfloor \frac{c_{min} + 28}{10} \right\rfloor , \left\lfloor \frac{c_{max} + 28}{10} \right\rfloor \}$$

with $c_{max} = 3$

$$c_2^h \in \{ \left\lfloor \frac{28}{10} \right\rfloor , \left\lfloor \frac{31}{10} \right\rfloor \} = \{2, 3\}$$

therefore $c_2^h$ is chosen uniformly at random between 2 and 3. The heuristic thus fails in solving **186 + 261 + 198 + 256** with a chance of 50%.

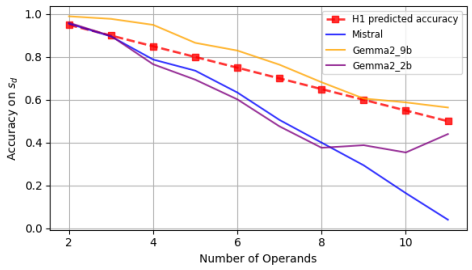



Figure 7: Accuracy of first generated result digit $s_d$ in zero-shot multi-operand addition for Mistral and Gemma-2 variants, compared to the expected accuracy based on **H1**.

For 4-operand addition, there are 37 possible sums for the second digits (ranging from 0 to 36). In 28 out of these 37 cases, the heuristic reliably determines the correct carry bit. However, when $t_1 \in \{7, 8, 9, 17, 18, 19, 27, 28, 29\}$, the heuristic must randomly choose between two possible carry values, leading to a 50% chance of selecting the correct one. This results in an overall accuracy of:

$$\frac{28 \times 1.0 + 9 \times 0.5}{37} = 0.878$$

Thus, the heuristic only achieves 88% accuracy in correctly predicting the first result digit $s_2$ in 4-operand addition, compared to the 97% accuracy in two-operand addition. In Appendix E, we provide exact values for $s_2$ accuracy as predicted by **H1**, for addition tasks between 2 and 11 operands.

### 6.2 Empricial Evidence on Multi-Operand Addition

Intuitively, according to **H1**, Mistral and Gemma-2 with their one-digit tokenization should fail at multi-operand addition at a certain rate: The amount of instances in which a lookahead of one digit is sufficient to accurately predict $s_i$ gets smaller and smaller because the carry bit value can get larger and larger for multiple operands. We test if **H1** holds in predicting the first generated digit $s_d$ in Mistral and Gemma-2 for multiple operands. We evaluate prediction accuracy on the multi-operand datasets described in Section 2.1. **H1** should provide an upper bound for the performance of LLMs[5] for predicting the first result digit $s_d$. Figure 7 shows that **H1** is indeed a very precise upper bound for the accuracy of the zero-shot[6] generation of the first result digit $s_d$ by Mistral and Gemma-2. We take this as further evidence that these LLMs make use of **H1**. Models may exhibit additional difficulties beyond the one-digit lookahead limitation, explaining their underperformance on calculations with many operands (8-10). We suspect this discrepancy may be related to limited training exposure to these many-operand addition tasks, but further investigation is needed to confirm this.

[5]Autoregressive LLMs with single-digit tokenization of numbers.

[6]Results for the one-shot setting are in Appendix D.

## 7 Multi-Digit Tokenization Models Employ the Same Heuristic

While Levy and Geva (2024) demonstrate that all LLMs, regardless of the tokenization strategy, internally represent numbers as individual digits, it remained unclear whether models with multi-digit tokenization also rely on a one-digit lookahead when generating addition results. In this section, we show that perhaps surprisingly multi-digit tokenization models, such as Llama-3, also employ a lookahead of one **digit** when predicting carry bits. To show this, we design 3 controlled datasets that force the multi-digit tokenization model Llama-3 to generate results across multiple tokens. To investigate the effect of model size, we also include results on the 70B version of LLama-3.

**Experimental Setup.** To examine whether Llama-3 employs a one-digit lookahead, we use six-digit numbers in two-operand addition (e.g., "231234 + 124514 = "), where each operand is tokenized into two three-digit tokens by the model's tokenizer, such as: [" 231"," 234", " +", " 124", " 514", " ="] and the result is generated as two triple-digit tokens as well, in this example [" 355", " 748"]. The first generated triple-digit token $s_5s_4s_3$ corresponds to digit base positions $10^5$, $10^4$, and $10^3$. If Llama-3 did employ **H1** it would look ahead to digit position $10^2$, but ignore digit positions $10^1$ and $10^0$, as they fall outside the lookahead window.

**Carry Scenarios.** We evaluate model behavior in three datasets with six-digit operands (ranging from 100,000 to 899,999) and results between 200,000 and 999,999. We use a zero-shot prompt template. Each dataset consist of 100 samples:

- **DS6: No carry.** The addition does not produce any carry and no digits sum to 9. (e.g., $111,234 + 111,514 = 222,748$).
- **DS7: Direct carry in position $10^2$.** A carry is generated at $10^2$ and directly affects $10^3$ (e.g., $111,721 + 111,435 = 223,156$).

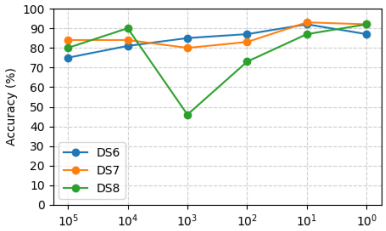



Figure 8: Per-digit generation accuracy of Llama on datasets DS6-DS8.

- **DS8: Cascading carry from $10^1$ to $10^3$.** A carry originates at $10^1$, cascades to $10^2$ and then affects $10^3$ (e.g., $111,382 + 111,634 = 223,016$).

**Expected Outcomes.** If Llama-3 employs **H1**, we expect that DS6 should be easy, as no carry propagation is required. DS7 should also be easy, since the carry affecting $10^3$ is within the one-digit lookahead window. DS8 in contrast should be challenging, as the carry originates from $10^1$, from beyond the model's lookahead range. We expect a lower accuracy in generating $10^3$, the result digit that is affected by the potentially inaccurate carry.

**Results.** Figure 8 shows that Llama-3 exhibits the expected pattern predicted by **H1**. The sharp drop in accuracy in dataset DS8 on digit $10^3$ provides evidence that Llama-3, regardless of its multi-digit tokenization strategy, relies on the same one-digit lookahead for solving addition left to right. Results for LLama-3-70-B are given in Appendix F and show the exactly same behaviour, which indicates that **H1** is employed irrespective of model size.

## 8 Direct Carry Circuit

Building on **H1**, which posits that LLMs rely on a single-digit lookahead to anticipate carries, we investigate the mechanistic implementation of this heuristic in Gemma-2-9B. Following Baeumel et al. (2025), who demonstrate that LLMs use modular, digit-specific circuits to generate result digits independently, we find that these models contain a circuit for the *next* digit—the one not yet generated. This circuit acts as a one-digit lookahead, signaling whether a direct carry is required.

Baeumel et al. (2025) identify a clear hundreds ($s_2$) circuit in Gemma-2-9B responsible for generating the $s_2$ digit (i.e., the 'hundreds' digit, Figure 9) and also report a less dominant tens ($s_1$) circuit in intermediate MLP layers. In contrast, no dedicated units ($s_0$) circuit exists. We test whether the presence of the $s_1$ circuit, together with the absence of an $s_0$ circuit, implements the one-digit lookahead heuristic (**H1**): the $s_1$ circuit may signal whether the next digit generates a carry that directly affects the current generation.

### 8.1 Carry Intervention

We reuse the digit-position-specific neurons released by Baeumel et al. (2025) to test whether the tens ($s_1$) circuit implements the single-digit lookahead, functioning as a direct carry circuit.

**Data and Method.** We create 200 intervention data samples to intervene on the tens-digit circuit of a no-carry base prompt ($347 + 231 = 578$) with a source prompt that causes a carry from tens to hundreds ($347 + 482 = 829$). We compare whether after the intervention the model indeed outputs a carry adjusted hundreds digit (i.e., $6 = b_{+1}$) instead of the base hundreds digit (i.e., $5 = b$). We intervene on the tens circuit of Gemma 2 9B (layers 28-31) intervention data. This setup allows us to determine whether carry information is localized within the tens circuit or processed elsewhere in the model.

The results match our expectation[7]: the probability of $b$ decreases by 49.6 percentage points, while the probability of $b_{+1}$ increases by 48.8 percentage points. This demonstrates that the tens circuit indeed transmits carry information to influence the current generation step.

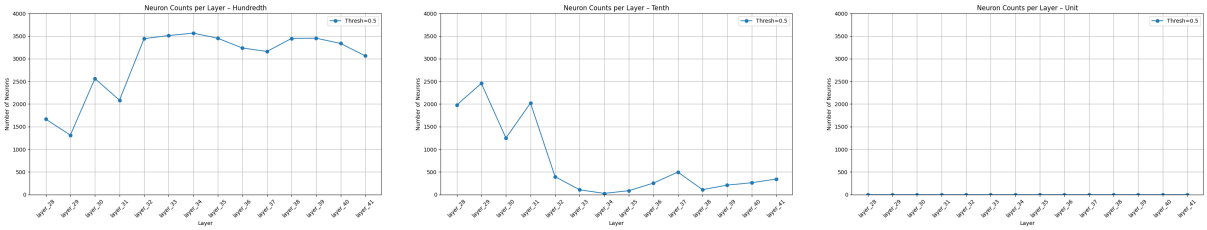
We thus establish that the tens circuit functions mechanistically as a direct carry circuit, implementing the one-digit lookahead.

## 9 Related Work

Recent work has benchmarked the arithmetic capabilities of LLMs using text-based evaluations and handcrafted tests (Yuan et al., 2023; Lightman et al., 2023; Frieder et al., 2023; Zhuang et al., 2023). Numerous studies consistently show that LLMs struggle with arithmetic tasks (Nogueira et al., 2021; Qian et al., 2022; Dziri et al., 2023; Yu et al., 2024).

Zhou et al. (2023) and Zhou et al. (2024a) examine transformers' ability to learn algorithmic procedures and find challenges in length generalization (Anil et al., 2022). Similarly, Xiao and Liu (2024) propose a theoretical explanation for LLMs' difficulties with length generalization in arithmetic.

[7]We choose the following optimal thresholds for neuron circuit membership $t* = 0.5$




(a) Hundreds (b) Tens (c) Unit

Figure 9: *Circuit Size*: Number of MLP neurons per layer in digit position circuits (threshold for Fisher Score $t* = 0.5$. Dimensionality of MLP updates in Gemma 2 9B is 3584.

Gambardella et al. (2024) find that LLMs can reliably predict the first digit in multiplication but struggle with subsequent digits.

The focus of research has recently shifted from mere benchmarking of LLMs to trying to understand *why* LLMs struggle with arithmetic reasoning. Using circuit analysis, Stolfo et al. (2023) and Hanna et al. (2023) explore internal processing in arithmetic tasks, while Nikankin et al. (2024) reveal that LLMs use a variety of heuristics managed by circuits. In contrast, Baeumel et al. (2025), Quirke and Barez (2023) and Deng et al. (2024) show that models process arithmetic with structured and algorithmic approaches rather than simple memorization, through pattern recognition or digit-wise, position-specific streams. Kantamneni and Tegmark (2025) showed that LLMs represent numbers as generalized helixes and perform addition using a "Clock" algorithm (Nanda et al., 2023). Levy and Geva (2024) demonstrate that numbers are represented digit-by-digit, extending Gould et al. (2023), who find that LLMs encode numeric values modulo 10. Zhu et al. (2025) suggest that numbers are encoded linearly, while Marjieh et al. (2025) indicate that number representations blend string-like and numerical forms.

Another line of research explores how tokenization influences arithmetic capabilities. Garreth Lee and Wolf (2024) show that single-digit tokenization outperforms other methods in simple arithmetic tasks. Singh and Strouse (2024) highlight that right-to-left (R2L) tokenization—where tokens are right-aligned—improves arithmetic performance. Zhou et al. (2024b) investigate the influence of the choice of numeral system on arithmetic capabilities. Notably, they finetune LLMs on addition tasks, which differs from our work in that such LLMs are more specialized and not evaluated off-the-shelf.

In a similar spirit, a strand of work investigates arithmetic skils of LLMs after finetuning or adapting them. The role of embeddings and positional encodings is emphasized by McLeish et al. (2024), who demonstrate that suitable embeddings enable transformers to learn arithmetic, and by Shen et al. (2023), who show that positional encoding improves arithmetic performance. Zhang-Li et al. (2024) show that reversing the order of calculations and forcing an LLM to work right-to-left improves arithmetic performance. Chen et al. (2024) investigate underlying computation mechanisms in multi-operand addition, similar to our work, but train models on arithmetics; and they restrict themselves to two-digit operands.

## 10 Conclusion

Based on the evaluation of several state-of-the art models from different families, our study indicates that pretrained LLMs, regardless of their numeric tokenization strategy and model size, rely on a simple one-digit lookahead heuristic for anticipating carries when performing addition tasks. While this strategy is fairly effective for two-operand additions, it fails for multi-operand additions due to the increasingly unpredictable value of cascading carry bits. Through probing experiments and targeted evaluations of digit-wise result accuracy, we demonstrate that model accuracy deteriorates precisely at the rate the heuristic predicts. Mechanistic analyses further reveal that this heuristic is implemented via specialized digit-position circuits, with dedicated pathways transmitting carry information to influence subsequent predictions.

Our findings reveal a core limitation in current LLMs: Their difficulty generalizing to complex arithmetic tasks. This work deepens understanding of their reasoning limits and points to increased lookahead as a promising way to improve performance on such tasks.

## Limitations

Our work highlights limited lookahead as a key challenge for LLMs when adding multiple numbers. However, it remains unclear whether this limitation extends to other arithmetic operations, such as subtraction. Additionally, we cannot determine whether the limited lookahead is a heuristic explicitly learned for arithmetic tasks, or if it could also affect general language generation tasks as thus hinder performance of other tasks that require long-range dependencies. Future work should explore the depth of lookahead in tasks beyond arithmetic.

While the lookahead heuristic offers a straightforward explanation for the upper performance limit of LLMs on addition, it does not fully account for why LLMs still somewhat underperform relative to the heuristic in addition tasks with many operands (e.g., adding 8–11 numbers). We suspect this discrepancy may be related to limited training exposure to these many-operand addition tasks, but further investigation is needed to confirm this.

Finally, we do not tackle methods to overcome the shallow lookahead. Future work should investigate whether targeted training on tasks requiring deeper lookahead can encourage models to deepen their lookahead.

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## A Example Addition According to Formalization

We show a concrete example for two-operand addition according to the formalization defined in Section 4. For $147 + 255$, we have:

$k = 2, d = 3, n1 = [1, 4, 7], n2 = [2, 5, 5]$.

We then compute:

$$T_2 = c_2 + 1 + 2$$
$$T_1 = c_1 + 4 + 5$$
$$T_0 = c_0 + 7 + 5 = 0 + 7 + 5 = 12$$
$$s_0 = 12 \mod 10 = 2, \quad c_1 = \left\lfloor \frac{12}{10} \right\rfloor = 1$$
$$T_1 = 1 + 4 + 5 = 10$$
$$s_1 = 10 \mod 10 = 0, \quad c_2 = \left\lfloor \frac{10}{10} \right\rfloor = 1$$
$$T_2 = 1 + 1 + 2 = 4$$
$$s_2 = 4 \mod 10 = 4, \quad c_3 = \left\lfloor \frac{4}{10} \right\rfloor = 0$$
$$S = [0, 4, 0, 2]$$

The result of the addition is 402.

## B Generation Accuracies for 2-Operand, 3-Digit Addition

We show the generation accuracy of the full result $S$ and the digit-wise accuracy of $s_2$, compared across the different carry bit datasets, as referenced in Section 4. Table 1 shows that Gemma-2 and Mistral struggle with the generation of the correct result digit $s_2$, exactly in the datasets that **H1** predicts to be difficult. DS3 and DS5 contain addition tasks in which a lookahead of one digit is insufficient ot determine the value of $s_2$.

| | | DS1 | DS2 | DS3 | DS4 | DS5 |
|---|---|---|---|---|---|---|
| | $c_2^h = ...$ | 0 | 0 | ? | 1 | ? |
| **S** | **Mistral** | 0.99 | 1.00 | 0.77 | 1.00 | 0.71 |
| | **Gemma-2-9B** | 1.00 | 0.99 | 0.94 | 1.00 | 0.80 |
| | **Gemma-2-2B** | 0.96 | 0.93 | 0.68 | 0.92 | 0.74 |
| | **Llama-3** | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |
| $\mathbf{s_2}$ | **Mistral** | 1.00 | 1.00 | **0.77** | 1.00 | **0.71** |
| | **Gemma-2-9B** | 1.00 | 0.99 | **0.94** | 1.00 | **0.80** |
| | **Gemma-2-2B** | 0.98 | 0.96 | **0.68** | 0.99 | **0.75** |
| | **Llama-3** | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |

Table 1: Generation accuracy of the full result $S$ and the digit-wise accuracy of $s_2$, compared across the different carry bit datasets.

## C Example: H1 Failure on 4-Operand Addition

Below is an example in which the heuristic **H1** fails in 4-operand addition, visualized in Figure 10: **186 + 261 + 198 + 256.**

$$t_1 = 8 + 6 + 9 + 5 = 28$$
$$c_2^h \in \{ \left\lfloor \frac{c_{min} + 28}{10} \right\rfloor, \left\lfloor \frac{c_{max} + 28}{10} \right\rfloor \}$$

with $c_{max} = 3$

$$c_2^h \in \{ \left\lfloor \frac{28}{10} \right\rfloor, \left\lfloor \frac{31}{10} \right\rfloor \} = \{2, 3\}$$

therefore $c_2^h$ is chosen uniformly at random between 2 and 3. The heuristic thus fails in solving **186 + 261 + 198 + 256** with a chance of 50%.

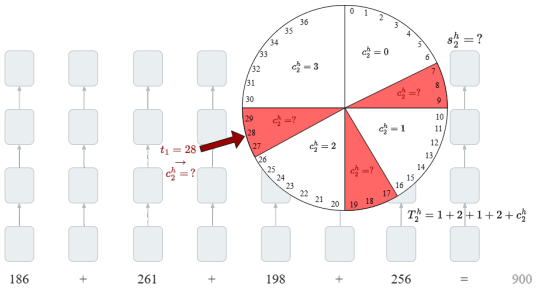


Figure 10: 4-operand addition in which **H1** fails.

| Nr. Operands $k$ | $c_{max}(k)$ | Values of $t_i$ in which H1 fails | Expected acc. on $s_d$ |
|---|---|---|---|
| 2 | 1 | 1 fail:= 9 | $\frac{18\times1.0+1\times0.5}{19} = 0.974$ |
| 3 | 2 | 4 fails:= 8, 9, 18, 19 | $\frac{24\times1.0+4\times0.5}{28} = 0.928$ |
| 4 | 3 | 9 fails:= 7, 8, 9, 17, 18, 19, 27, 28, 29 | $\frac{28\times1.0+9\times0.5}{37} = 0.878$ |
| 5 | 4 | 16 fails:= 6, 7, 8, 9, 16, ..., 39 | $\frac{30\times1.0+16\times0.5}{46} = 0.826$ |
| 6 | 5 | 25 fails:= 5, 6, 7, 8, 9, 15, ..., 49 | $\frac{30\times1.0+25\times0.5}{55} = 0.773$ |
| 7 | 6 | 36 fails:= 4, 5, 6, ..., 59 | $\frac{28\times1.0+36\times0.5}{64} = 0.719$ |
| 8 | 7 | 49 fails:= 3, 4, 5, ..., 69 | $\frac{24\times1.0+49\times0.5}{73} = 0.664$ |
| 9 | 8 | 64 fails:= 2, 3, 4, ..., 79 | $\frac{18\times1.0+64\times0.5}{82} = 0.610$ |
| 10 | 9 | 81 fails:= 1, 2, 3, ..., 89 | $\frac{10\times1.0+81\times0.5}{91} = 0.555$ |
| 11 | 9 | 89 fails:= 1, 2, 3, ..., 99 | $\frac{10\times1.0+90\times0.5}{100} = 0.540$ |

Table 2: Predicted accuracy on the first result digit $s_d$ in the addition of multiple numbers according to **H1**.

## D Zero-shot Generation Accuracy

We test if **H1** holds up in predicting the generation accuracy on $s_d$ of Mistral and Gemma for multiple operands. Figure 11 shows that **H1** provides an upper bound for the generation accuracy of $s_d$ in a one-shot setting for Mistral and Gemma-2 on $s_d$.

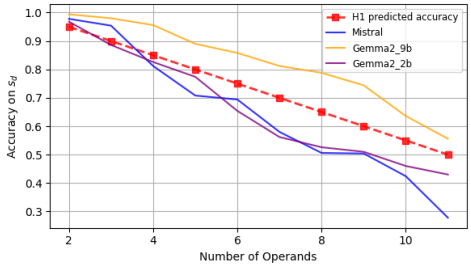


Figure 11: Accuracy of first generated result digit $s_d$ in one-shot multi-operand addition tasks for Mistral and Gemma-2 variants, compared to the expected accuracy on $s_d$ based on **H1**.

## E Accuracy Prediction of Heuristic

Table 2 contains, for addition tasks with different numbers of operands $k$, the maximum value of the carry $c_{max}(k)$. Based on $c_max$ it list those values of $t_i$ in which **H1** is insufficient to accurately predict $s_2$.

## F Results on DS6-8 for larger models

Table 3 shows that Llama-3-70B also employs H1.

| **Data Set** | $10^5$ | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ |
|---|---|---|---|---|---|---|
| **DS6** | 0.70 | 0.77 | 0.79 | 0.76 | 0.77 | 0.79 |
| **DS7** | 0.66 | 0.73 | 0.67 | 0.72 | 0.78 | 0.76 |
| **DS8** | 0.65 | 0.72 | **0.48** | 0.63 | 0.69 | 0.74 |

Table 3: Results for LLama-3-70B on Data Sets 6-8. The expected lowest accuracy of the model for $10^3$ on DS8 is **bold printed**.