

# T-FREE: Subword Tokenizer-Free Generative LLMs via Sparse Representations for Memory-Efficient Embeddings

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## Abstract

Tokenizers are crucial for encoding information in Large Language Models, but their development has recently stagnated, and they contain inherent weaknesses. Major limitations include computational overhead, ineffective vocabulary use, and unnecessarily large embedding and head layers. Additionally, their performance is biased towards a reference corpus, leading to reduced effectiveness for underrepresented languages. To remedy these issues, we propose T-FREE which directly embeds words through sparse activation patterns over character triplets, and does not require a reference corpus. T-FREE inherently exploits morphological similarities and allows for strong compression of embedding layers. In our exhaustive experimental evaluation, we achieve competitive downstream performance with a parameter reduction of more than 85% on these layers. Further, T-FREE shows significant improvements in cross-lingual transfer learning.

## 1 From Text Representations For Machine Learning

Large language models (LLMs) have shown remarkable abilities in processing natural language and various data types. The tokenizer, an essential part of any language-based LLM, splits input text into subwords and converts textual data into integer representation. It is built by populating a fixed-size vocabulary based on statistical frequencies in a reference corpus (Sennrich, 2016; Kudo and Richardson, 2018). With the LLM’s trained embedding layers, these integers are converted into floating-point representations (Mikolov et al., 2013b; Press and Wolf, 2017; Vaswani et al., 2017). These components significantly shape the training objectives and influence what an LLM can process, interpret, and generate. Despite advances, the basic principles of tokenization and embeddings have remained

largely unchanged in recent years.

Although this approach has served the LLM community well, and influential characters target to tokenize all kinds of data to “lead a new industrial revolution”<sup>1</sup>, it has significant inherent weaknesses. For one, subword tokenizers require dedicated training and, as such, additional computational resources. Design choices and errors at this stage can negatively impact the downstream model (Ali et al., 2023). Any tokenizer’s vocabulary is heavily optimized for the reference corpus, leading to strong drops in performance for, e.g., underrepresented languages. We also show that the resulting vocabulary of subword tokenizers is poorly utilized, where up to 34% of tokens are near duplicates with limited additional information. Despite that, the corresponding embeddings are trained independently. These issues have caused a significant expansion in the size of vocabularies and their corresponding embedding layers, with billions of parameters being allocated exclusively for text encoding and decoding.

To remedy these issues and challenge the traditional views, we propose a paradigm shift on how we embed and decode text for LLMs. We present tokenizer-free sparse representations for memory-efficient embeddings (T-FREE) as an alternative to subword tokenizers. We directly embed each word in the input text with sparse activation patterns over hashed character triplets. Consequently, we eliminate the need for subword tokens, thus retaining near-optimal performance across languages. Additionally, T-FREE explicitly models character overlaps between morphologically similar words without the need to learn an embedding for each variant from scratch through a one-to-one bijection. The backbone of the language model will remain free of subword tokenization as we directly

<https://github.com/Aleph-Alpha/trigrams>

<sup>1</sup>[https://x.com/tsarnick/status/1801884651986030820?s=12&t=5I\\_](https://x.com/tsarnick/status/1801884651986030820?s=12&t=5I_)

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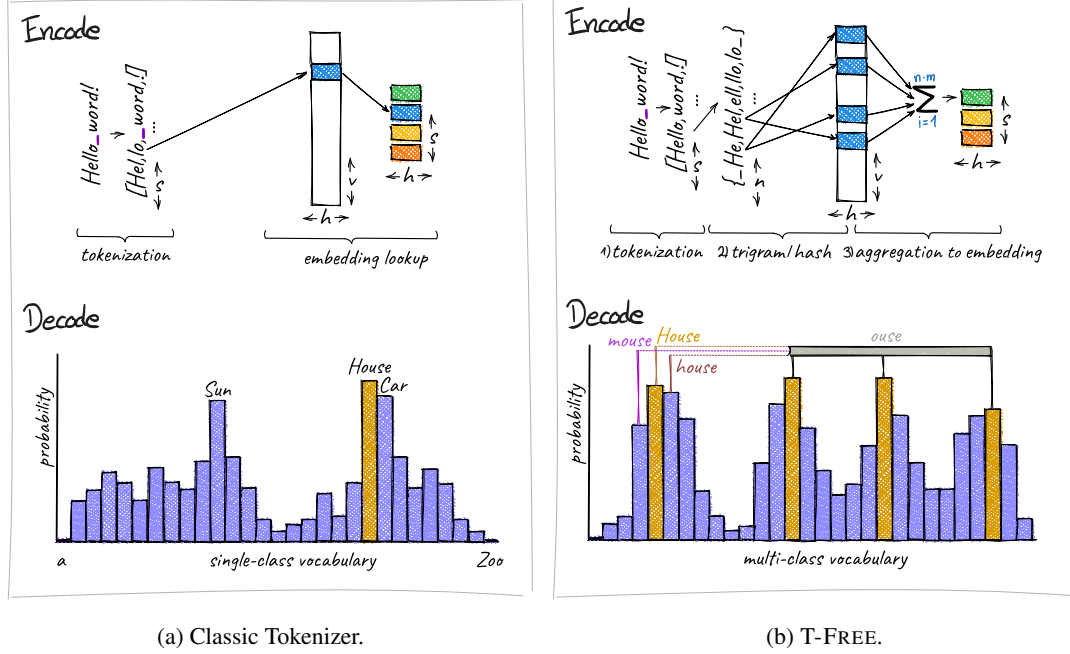


Figure 1: Method comparison of classic Tokenization (left) and T-FREE (right) for text encoding (top) and decoding (bottom). Classic subword tokenizers learn a single-label vocabulary, i.e. a token is bijectively mapped into a single entry of the vocabulary. Instead, T-FREE uses a bijective multi-label mapping over multiple activations of hashed character trigrams. As T-FREE explicitly models morphological similarities, it enables compression of the embedding layer.

encode the textual representation. We argue that the converged embedding of such similar words should remain close and, thus, can be heavily compressed<sup>2</sup>. This exploitation of similarities allows T-FREE to reduce the size of the embedding layers by 87.5%<sup>3</sup> and the average encoding length of text by 56%<sup>4</sup>. In addition to the inherent benefits of T-FREE, the approach remains highly competitive on standard downstream model performance benchmarks. Finally, for transfer learning to an unseen language, the T-FREE model quickly improves performance, while the tokenizer baseline shows only minor adaptation.

Our contributions can be summarized as follows:

- We systematically demonstrate the inherent weaknesses of common tokenization and embedding approaches.
- We propose T-FREE, a powerful and efficient alternative for tokenizer-free LLMs.
- We exhaustively evaluate hyperparameters of T-FREE on established benchmarks by train-

ing 1B LLMs from scratch. Our comparison against equally trained models with classic tokenization demonstrates competitive performance despite the significant reduction in compute resources and parameters.

- We demonstrate the capabilities of T-FREE for cross-lingual transfer on continual pre-training on a 3B LLM.

## 2 Classic Tokenization Principles

Before we derive T-FREE in detail, let us first establish some basics of how LLMs traditionally encode and decode text. Most LLM operations are performed in floating-point numbers through a series of matrix multiplications and non-linear activation functions. Consequently, we require techniques that map discrete textual inputs into floating-point representations and inversely transform the predictions of the model back to text.

Traditionally, the first step in this process is to split any textual input into small chunks referred to as *tokens*. Generally, these tokens can take arbitrary formats, spanning numerous characters, a single or even multiple words, and may also contain special characters. The latter can be particularly useful to encode programming languages. A *tokenizer* com-

<sup>2</sup>The English language contains about 500k words, while “native fluency” is achieved at 10k words (Nation, 2006).

<sup>3</sup>Compared to our 64k unigram baseline.

<sup>4</sup>Compared to Mistral 32k avg. of EN, DE, RU, VI, AR.

prises the steps and rules that are necessary to dissect a text into a sequence of tokens. Importantly, the total number of tokens is restricted, and we refer to the set of all unique tokens as the *vocabulary*.

Each token in the vocabulary is assigned an integer *token-id*, wherefore tokenizers produce a sequence of token-ids for any textual input. Next, a large matrix of dimensions *vocab size*  $\times$  *hidden size*, an LLM’s *embedding layer*, maps each token-id to an internal representation as a floating point vector (cf. Fig. 1a). To produce new text, generative models are trained auto-regressively. That is, they iteratively predict the next token, which is appended to the input text. Therefore, the training objective is formulated as a classification problem: a one-label prediction of the next token over the entire vocabulary. Consequently, the last layer of the model—the *LM head*—is a projection into the size of the vocabulary and thus also of dimension *vocab size*  $\times$  *hidden size*. For decoding, we can, for example, always select the token with the highest assigned value, which is called *greedy sampling*. The output text is produced by looking up the corresponding text snippet of each predicted token-id in the vocabulary.

Generally, it is desirable to encode any text in as few tokens as possible to reduce computational cost. At the same time, different semantic concepts should be separated into distinct tokens to ensure good language comprehension. The combination of both objectives is usually best satisfied by encoding each word as one token.

## 2.1 Tokenizer Algorithms

The vast majority of LLMs utilize a tokenizer built with one of two approaches. Both progressively build up tokenization rules and their vocabulary based on statistics in a reference corpus.

**Byte Pair Encoding (BPE).** BPE (Sennrich, 2016) starts with a set of all characters as individual tokens. Progressively, the most frequent token pairs occurring together in the training documents are merged. The resulting new token and the merging rule are added, and the training is completed when the desired number of tokens is reached.

In order to encode text with the trained tokenizer, BPE splits the input into individual characters and applies the lowest-ranking merge rule until no more are applicable. This exhaustive search can become computationally intensive, especially for long input sequences and large vocabularies.

**Unigram.** Unigram (Kudo and Richardson,

2018) operates inversely to BPE. First, it splits the training corpus into a large set of reference words and their respective frequencies. The vocabulary is initially populated with all possible substrings of these words. At each iteration, Unigram computes a loss of the current vocabulary with respect to the training corpus for all possible tokenizations. The least influential tokens are then removed until the desired vocabulary size is reached. For text encoding, the Viterbi algorithm is applied to determine the most preferred segmentation of a given word based on the ranked available tokens.

The text decoding in both cases maps directly back into the vocabulary list and the respective sub-words. To ensure that every word can be represented, a “byte-fallback” into unicode is often used for characters not present in the vocabulary.

## 2.2 Facing the Flaws

Common to both methods is a set of distinct flaws.

**Large Vocabularies F1)** Words that do not appear in the vocabulary are split into multiple tokens and, as such, require more compute during model inference and training. To avoid out-of-vocabulary words and to achieve the best downstream representations on a diverse set of languages and tasks, researchers tend to use ever larger vocabularies. Although some models still rely on a  $32k$  vocabulary (Touvron et al., 2023; Jiang et al., 2023), more recent releases go up to  $128k$  (Meta, 2024) or even beyond  $250k$  (Mesnard et al., 2024; Gomez, 2024). Large vocabularies, in turn, require large embedding and head layers. For example, Command-R (Gomez, 2024) with a hidden dimension of 12,288 and a vocabulary of 256,000 tokens uses  $6.3B$  parameters only for the embedding and head layer. Naturally, a large number of parameters complicate model training and may require advanced sharding techniques such as “model parallelism”. Even the tokenization itself can become (CPU-) computationally intense for large documents and vocabularies. Naturally, embedding matrices of this scale are generally not an option for smaller “on-the-edge” models. Nevertheless, they still occupy a large portion of parameters in smaller models, e.g. 40% for Gemma-2B (Mesnard et al., 2024).

**Duplicate Tokens F2)** Furthermore, the allocated vocabulary is expected to be poorly utilized due to the statistically likely occurrence of near-duplicate tokens. Most prominently, a significant portion of tokens appears multiple times, only differing in capitalization or the existence of a lead-

ing whitespace (*cf.* Sec 4.3). For example, to spell all 64 substrings and variations of the word “\_words”<sup>5</sup>, we require a total of 37 unique tokens (*cf.* App. Tab. 7). Since the corresponding embeddings of all tokens are independent and randomly initialized, the representation of each duplicate token needs to be learned from scratch without exploiting morphological synergies. Further, large embedding layers are purely utilized since some tokens will rarely occur. The corresponding embedding weights of these tokens are thus seldom active while still requiring compute.

**Training data overfitting F3)** As discussed above, these tokenizers require dedicated training before the actual model training. In addition to the added computational overhead, the data selection and potential mistakes during tokenizer training have significant impact on the subsequent LLM (Ali et al., 2023). For natural language, for example, this paradigm may result in a vocabulary tailored to one language (usually English) and consequently drops in performance for others, especially those not explicitly included. The resulting LLM may still be somewhat adapted to other languages since many similar low-level structures (Mikolov et al., 2013a). However, its overall training and inference performance will not be as efficient as we demonstrate.

In contrast, T-FREE addresses all of these disadvantages. It is computationally efficient and performs good tokenization across languages without duplicates. It drastically reduces the parameters required for text encoding, exploiting word spelling similarities. Importantly, none of these improvements sacrifices downstream model performance.

### 3 T-FREE

A key motivation for T-FREE is the intuition that minor differences in spelling, like leading whitespaces or capitalization, do not hold enough entropy to justify entirely independent tokens. T-FREE directly encodes morphological similarities by representing each word as a multi-label encoding of its character triplets. This designed overlap between words allows us to significantly reduce the size of embedding layers.

We now derive T-FREE’s approach to text encoding and decoding and discuss implications on LLMs in general. We provide a visualization of each step in Fig. 1b and pseudo-code in App. A.

<sup>5</sup> \_ represents a whitespace.

### 3.1 Text Encoding

**Step 1: Word splitting.** First, we rigorously split the text by digits and non-alphanumeric characters. The resulting splits, therefore, contain entire words, digits, or special characters. We consider each digit separately, as it is standard in SOTA LLMs (*cf.* Tab. 1). Specifically, we include the 10 digits 0 to 9, and otherwise, we rely on attention to comprehend larger numbers or mixtures with characters.

By definition, we represent each word with a prefixed and suffixed whitespace. In particular, we assume that an entire word is encoded into a single embedding, and analogously, we predict an entire word at once. Consequently, we no longer need to explicitly model whitespace as a character and eliminate near-duplicate tokens. Nonetheless, we add a dedicated “whitespace” and “non-whitespace” token to the tokenizer. These special tokens allow us to model cases where substrings should (not) be concatenated with whitespace, e.g., single digits of larger numbers. To reduce their need, we further add a rule-set that favors (non-)whitespace in front or after certain characters. Generally, we prefer to add no whitespace after a digit embedding and similarly no whitespace before punctuation. A detailed description of the rule set is found in App. B.

Considering the example in Fig. 1b, the input text “Hello\_word!” would be tokenized as [‘Hello’, ‘word’, ‘!’].

**Step 2: Encoding.** Next, we define a robust hash function that uniformly encodes a token into  $n$  descriptors, where  $n$  usually equals the word-length<sup>6</sup>. Specifically, we apply convolutions of size three and byte-wise stride to each word. This operation yields a set of character triplets, which we refer to as “trigrams”. Consequently, “Hello” is decomposed into {\_He, Hel, ell, llo, lo\_}. Trigrams usually contain enough information about the relationship between letters to reassemble the word from the unordered set.

Subsequently, we project each trigram descriptor into a sparse hidden representation vector of  $m$  “active entries” on the embedding layer. Specifically, T-FREE calculates  $m$  numerical hashes of each trigram, which can be considered as identifiers. We map these into the LLMs embedding matrix by calculating each hash value modulo  $v$  to identify the active indices. The selection of vocab size  $v$  is further explained in Step 3.

Overall, we obtain  $n \cdot m$  total activations for any

<sup>6</sup>Only exceptions are unicode fallbacks.



single word. To further exploit word similarities and bootstrap training, we calculate  $k \in [0, m)$  out of these hash calculations with the lowercased trigram. This mapping from trigram to hidden representation is static and can be precomputed<sup>7</sup>.

**Step 3: Aggregation.** Similar to classic embedding approaches (cf. Fig. 1a) T-FREE also utilizes an embedding matrix of dimension  $v$  with hidden size  $h$ . However, we do not have a fixed vocabulary, whose size dictates  $v$ . Instead, we can independently choose  $v$  as a hyperparameter with words and trigrams sharing individual entries to better encode similarities. Lastly, we sum all  $n \cdot m$  embedding entries to produce the final one embedding corresponding to a word, such as “Hello”.

Note again, that we utilize a significantly smaller number of embeddings than there are trigrams. While their hashes may naturally overlap, we ensure the uniqueness of the entire patterns through the  $m$  simultaneous hashes. As we ensure that trigram encodings do not collide, neither will the word encodings.

### 3.2 Training Objective & Text Decoding

As T-FREE’s representation of a word is now a multitude of activations, we reflect this change in the LM head, as well (cf. *Decode* sections in Fig. 1, App. Alg. 3,5). In particular, we change the target loss function from classic single-label binary cross-entropy (BCE) to a multi-label (ML) BCE over all  $n \cdot m$  activations of the next word targets:

$$\mathcal{L}_{BCE}^{ML} = - \sum_{j=1}^v [y_j \log(\hat{y}_j) + (1-y_j) \log(1-\hat{y}_j)],$$

for  $\hat{y}$  being the LM’s prediction and  $y$  the binary target vocab labels with  $\sum_{j=1}^v y_j = n \cdot m$ .

Next token decoding is shown in Fig. 2. We first assemble a dictionary of all possible next words and pre-compute their activation patterns. Importantly, only  $n \cdot m$  out of  $v$  entries will be non-zero for each word, and since we choose  $m \ll v$ , the dictionary matrix can be encoded as a sparse matrix, thus improving runtime. In addition, note the pattern similarity between similar words, as previously described. The last hidden layers’ output  $h$  is sigmoided, and multiplied with the dictionary matrix. Finally, we compute the average sigmoid value per dictionary entry,  $h'$ , to sample the next word, e.g. using standard argmax. Overall, for a dictionary with  $512k$  entries, this procedure only marginally increases the decoding runtime due to

the sparse property of the activation patterns. Further description, along with pseudocode, detailed depictions, and step-wise runtime analysis can be found in App. 5.

Note that the decode matrix is not required during training, and can dynamically be exchanged. We generate it by sampling the top-500k occurring words in the training dataset, and dynamically adding the missing words of the prompt.

### 3.3 Distinctions of paradigm shift

Notably, this paradigm shift to a multi-class vocabulary allows for more semantically robust decoding. With the classical approach, the distinctly noisy learning process can lead to unrelated concepts appearing among the top predictions (cf. ‘House’ and ‘Car’ in Fig. 1a). This effect can have a significant impact on next token sampling and potentially devastating outcomes for model modifications such as compression (Deiseroth et al., 2024). In contrast, the trigrammification and resulting embedding overlap of similar words with T-FREE inherently favors similar words during decoding (cf. ‘ouse’ in Fig. 1b). Moreover, activations in the embedding and LM head are more uniformly distributed, leading to better parameter utilization, and more stable model behavior.

The predictable words are still derived from a dictionary. However, this vocabulary list is exchangeable, and is not required during training. As such, depending on the use-case, it may be kept in reasonable sizes. Moreover a hierarchical decoding exploiting morphological structures can straightforward be implemented, e.g. first decoding lowercase words, and then uppercase variations (or similarly grouping by stems or endings).

Lastly, our design of a robust hash function on words addresses the afore mentioned flaws (Sec. 2.2) as the results of the next section demonstrate.

## 4 Empirical Evaluations

We continue with an empirical demonstration of the performance of T-FREE, and how it remedies the flaws of standard tokenizers as outlined in Sec. 2.2. To thoroughly analyze the performance differences, we designed three consecutive experiments: First, we perform hyperparameter ablations on a series of 1B parameter models, which achieve competitive scores on standard benchmarks with a reduced vocabulary, which in turn addresses F1. Second, we analyze the duplicates in the tokenizers of recent

<sup>7</sup>Note that there are only  $256^3 \approx 16.7M$  trigrams.

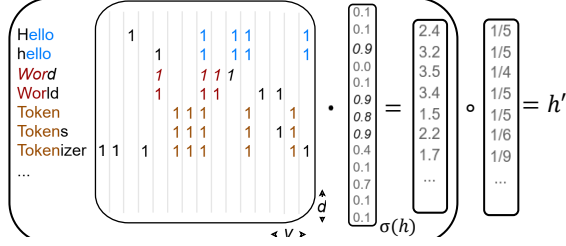


Figure 2: Example of the next word prediction with T-FREE. To the list of predictable words of dimension  $d$  we generate once the corresponding patterns within the available vocabulary size  $v$ , as described in the encoding step 2 of Sec. 3.1. Note how morphologically close words will generate overlapping patterns. The element-wise sigmoid values of the output of the last hidden layer,  $\sigma(h)$ , is multiplied with this pattern matrix using standard dot product. Finally, we use  $h'$  for the sampling process, the average sigmoid value of a word. C.f. App. A for further details.

LLMs with respect to **F2**. Notably, T-FREE is by design free of duplicates. Lastly, we look at **F3** and evaluate the performance of various tokenizers across languages. Further, we trained 3B parameter models on English and continued training on German data to practically investigate language adaptability. T-FREE has better tokenization performance across languages and outperforms classic tokenizers on language transfer.

#### 4.1 Experimental Details

First, let us clarify some details about our experimental setup. We provide more details for each section in the Appendix.

**Data and Code.** We use the slimpajama dataset (Soboleva et al., 2023) as our English and Occiglot Fineweb v0.5 (Brack et al., 2024) as our German data corpus. Both datasets contain a diverse range of content and have been extensively filtered and deduplicated.

As a baseline, we trained BPE and Unigram tokenizers of sizes  $32k$  and  $64k$  on a random 20GB slimpajama sample using Sentencepiece<sup>8</sup>. More details are described in App. C.

To ensure fair comparisons, we trained 1B and 3B models from scratch for the baselines and T-FREE using our adjusted code base<sup>9</sup>.

**LLM Pre-Training.** All models are transformer, decoder-only architectures similar to Llama-2. We solely change the tokenizer, embedding layer and LM head. Consequently, ablations with smaller

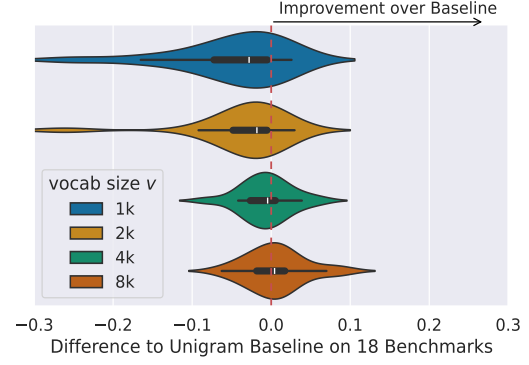


Figure 3: Hyperparameter search for Vocab Size of T-FREE on a series of 1B ablations. We fixed number of activations  $m = 10$ , and do not apply lowercase overlap ( $k = 0$ ). The boxplots show the differences of trained models to a  $64k$  unigram baseline for 18 downstream benchmarks (0-shot). T-FREE outperforms in median the classical tokenizer architecture with a reduced vocab size of  $8k$  entries (12.5%).

sizes of  $v$  result in a lower overall parameter count, heavily skewing the comparison in *favor of the baseline*. For hyper-parameter ablations of T-FREE, we train 1B models for  $50k$  steps with  $2k$  sequence length and  $1k$  total batch size. We then scale up the baseline and T-FREE models to 3B parameters and train for  $110k$  steps on slimpajama with  $4k$  sequence length. For the multilingual learning experiment, we continue training this English 3B model at a lower learning rate for another  $20k$  steps on German Occiglot data with a 20% replay of English.

**Evaluation.** We evaluate tokenizer performance in isolation using fertility measurements similar to Rust et al. (2021). Fertility benchmarks the number of tokens required per word with 1.0 thus being the optimal value. Specifically, we compare different tokenizers across 5 diverse languages on the respective data from Wikipedia.

Downstream benchmark comparisons are performed on 18 standardized benchmarks<sup>10</sup> in English that measure a wide variety of LLM capabilities, including general language modeling, question answering, and common sense reasoning. To evaluate german and english in comparison we use german translations of the Hellaswag, Truthfulqa and Arc-Challenge benchmarks<sup>11</sup>.

We built T-FREE’s prediction dictionary, from the top  $80k$  words that occurring in slimpajama, and additional top  $20k$  words from the German

<sup>8</sup><https://github.com/google/sentencepiece>

<sup>9</sup><https://github.com/Aleph-Alpha/trigrams>

<sup>10</sup><https://github.com/EleutherAI/lm-evaluation-harness>

<sup>11</sup><https://github.com/bjoernpl/GermanBenchmark>

Occiglot data.

#### 4.2 T-FREE performs at $8k$ vocab size

We present the results of our hyperparameter ablation study of T-FREE for  $1B$  models in Fig. 3. All scores are reported as differences to the Unigram  $64k$  baseline and for fixed parameters  $m = 10$  and  $k = 0$ . Generally, T-FREE remains highly competitive with the baseline as all versions outperform the Unigram model on some of the benchmarks. Further, we achieve the best results for a vocab size  $v$  of  $8k$  at which T-FREE outperforms the baseline on average. In contrast, a vocab size of  $\leq 2k$  seems insufficient with devastating outliers. We performed further ablations on parameters  $m$  and  $k$ , which are outlined in App. H.

These results demonstrate that T-FREE successfully addresses the flaw of large vocabularies and embedding layers (cf. F1 in Sec. 2.2). We are able to achieve competitive performance with only 12.5%<sup>12</sup> of the embedding parameters using T-FREE instead of Unigram.

Note, that we do not adjust any other model parameters when reducing vocab size. As such, the benchmark results compare a Unigram model with  $1.07B$  parameter against a T-FREE model with  $0.84B$  parameters (for  $v = 8k$ ). Consequently, we demonstrate that an LLM using T-FREE instead of Unigram performs better, despite having over 20% fewer parameters.

#### 4.3 T-FREE no duplicates by design

Let us now look into (near) duplicate tokens in commonly used tokenizers (cf. F2 in Sec. 2.2). In general, there are three types of overlaps in vocabularies: 1) The same token with and without capitalization, 2) with and without leading whitespace, and 3) dedicated tokens for multiple digits.

In Tab. 1, we report the percentage of duplicate tokens for our baseline tokenizers and commonly used models. Overall, between 15% and 35% of the available vocabulary is spent on (near) duplicate information with limited differences in entropy. Generally, tokenizers contain the most duplicates for capitalization, slightly fewer for whitespaces, and only a few duplicate digits. The relative amount of overlap tends to decrease with larger vocabularies, although Gemma marks an inglorious exception. In contrast, T-FREE is inherently designed to be free of duplicates. We can even adjust the param-

eter  $k$  to explicitly model the overlap of words to their lowercase representations. Consequently, all variants are inherently well represented in the embedding layer.

#### 4.4 T-FREE has lower fertility across, and is more adaptive to new languages

Finally, we investigate the versatility of tokenizers beyond their (main) language (cf. F3 in Sec. 2.2). We report the fertility of our baselines and other popular models in English, German, and three dissimilar languages that also contain significant character-level differences in Tab. 1. Common to all tokenizers is a significantly decreasing performance for non-English languages, especially for Russian and Vietnamese. Naturally, larger vocabulary sizes tend to have better multilingual coverage, in particular to language groups close to English, but still suffer from significant performance drops. In comparison, the tokenization of T-FREE, which is mainly based on whitespace splitting, provides comparably good performance across all 5 languages<sup>13</sup>. The increases in fertility for Russian or Vietnamese remain small and there is no performance difference for German or Arabic. Note that these synergies were explicitly modeled, and no reference corpus is needed to train and bias the fertility of T-FREE. Consequently, T-FREE allows for easier and more efficient model adaptation to low-resource languages.

We now explicitly show the devastating consequences of biased tokenizers on the language transfer capabilities of LLMs. As discussed above, we first train  $3B$  models for T-FREE and Unigram on English, and then transition to German. Through more ablations, we fixed the activations to  $m = 7$  and the lowercase trigram overlap to  $k = 3$ . Fig. 4 shows the performance average on the English and German versions of the standard benchmarks. The baseline performance in German is already improved with T-FREE, indicating that syntactic and semantic similarities between the languages are better captured in the learned representations. Additionally, T-FREE almost achieves the English-level performance on German after  $20k$  training steps. In contrast, the classical tokenizer variant improves only marginally with the same amount of training.

We, again, do not adjust any other model parameters when reducing the vocab size. As such,

<sup>12</sup> $8k$  instead of  $64k$ .

<sup>13</sup>More detailed evaluations are found in App. E.

Model/Tokenizer	Portion of duplicate tokens (%) ↓				Fertility across languages ↓				
	Total	Cap.	Space	Digits	EN	DE	RU	VI	AR
Unigram (64k)	35.24	23.27	13.47	0.00	1.280	2.004	11.431	5.060	9.455
BPE (64k)	35.24	23.27	13.47	0.00	1.275	2.025	11.423	4.755	9.465
Mistral (32k)	31.47	19.10	16.45	0.00	1.397	1.931	2.560	3.346	4.722
Phi-2 (50k)	23.23	12.91	16.89	3.32	1.265	2.266	6.729	4.339	5.225
Gemma (256k)	34.68	20.27	20.50	0.04	1.176	1.447	1.903	1.726	1.793
Command-R (255k)	15.31	15.31	14.00	0.00	<b>1.152</b>	1.411	1.590	1.597	1.578
<b>T-FREE (Ours)</b>	<b>0.00</b>	0.00	0.00	0.00	1.163	<b>1.182</b>	<b>1.338</b>	<b>1.400</b>	<b>1.086</b>

Table 1: Demonstration of inherent benefits of T-FREE over traditional tokenizers. The performance no longer degrades when confronted with languages beyond the one primarily trained on. Additionally, the vocabularies of classic tokenizers contain large portions of tokens only differing in their capitalization or leading whitespace. T-FREE does not construct such a vocabulary in the first place and thus utilizes embeddings more efficiently.

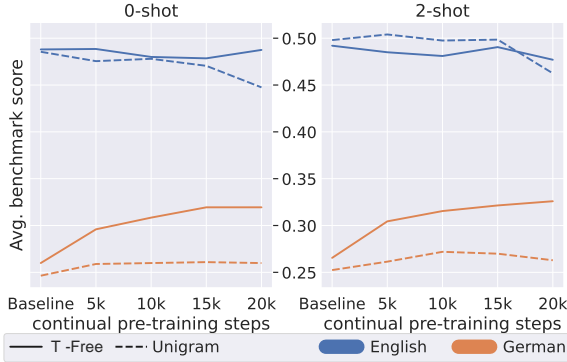


Figure 4: Continual pre-training performance. Trained are 3B models on English slimpajama data for 90k steps (“baseline”), and continued on German occiglot data for 20k steps. Plotted are the average scores of two benchmarks available in German and English: Hellaswag and Arc-Challenge. Notably, T-FREE outperforms in German already with the baseline. Within 20k continued steps, T-FREE improves by 5% on average in 0 and 2-shot, while the classic tokenizer approach barely improves. Both models slightly drop performance in English, albeit the tokenizer version more drastically. Full evaluations are found in Appendix Tab. 8,9,10.

T-FREE uses 10% fewer parameters than the baseline (2.77B instead of 3.11B) and still strongly outperforms the Unigram variant. More detailed evaluations are found in App. J.

## 5 Discussion

Prior research has demonstrated that the mapping into a sparse hidden representation and the training of a dense aggregation layer as applied in T-FREE, is a universal function approximator (Aved’yan, 1995). These results provide further theoretical motivation for our approach.

T-FREE allows for significant compression of an LLMs’ vocabulary by more than 85% without performance degradation. Notably, the affected

embedding and head layers are by far the largest in LLMs in terms of parameter count. They are also the most influential to an LLM, as they dictate the mapping between text and numerical representations. For one, these massive improvements allow for better utilization of billions of parameters in large models. The compression of T-FREE in particular paves the way to building better low-resource models, by reducing model size and training cost and improving adaptability. For example, in our experiments without pipe or model-parallelism, we were able to *triple* the micro-batch size, yielding faster training iterations.

Furthermore, we observed more stable loss curves for T-FREE, in particular for higher learning rates. These improvements may be attributed to the explicit modeling of similar words, the removal of duplicates, and the less volatile multi-label training target. Further, the uniform hashing distributes gradients evenly amongst the available vocab size, in contrast to classical approaches. We provide further details in App. D,G.

The rules we use for obtaining word representations are universal and well-defined at pre-training time. They do not change over time, particularly neither when adding languages later on. T-FREE also lowers computational costs due to its low fertility and easy-to-process whitespace splitting. Consequently, pre-processing, training and inference of an LLM all require less compute.

Lastly, T-FREE allows to explicitly model and steer the decoding process at inference time, by altering the available dictionary. Consequently, hallucinations will likely be reduced due to fewer “generic fall-back” word splits. Moreover, one can dynamically add or remove words. It is worth pointing out that T-FREE’s compression benefits can also be combined with traditional tokenizers. In-



stead of the simple whitespace splitting one could keep traditional tokenization and trigramify “classic tokens”.

## 6 Related Work

Few alternatives to BPE and Unigram have been found in recent LLMs and research. [Tay et al. \(2022\)](#) propose a gradient-based trainable tokenization module in contrast the otherwise statistical based approach.

The naive approach of splitting the input text into bytes or characters maximizes fertility and thus increases computational requirements. [Yu et al. \(2023\)](#) employ a mix of multiple models to improve this drawback of byte-wise processing. I.p. they introduce a fixed character-embedding aggregation and a second character-decoder model. However, they use a fixed byte-width that is processed at once, which is not aligned with word splits.

Consequently, prior research has proposed methods for merging bytes, e.g., through state-space models ([Wang et al., 2024](#)). However, these approaches still result in performance degradation. Finally, linguistically motivated approaches have built tokenizers based on known morphological rules ([Jabbar, 2024](#)). However, these methods are usually tailored to specific applications and are usually too costly and error-prone for large, general-purpose models.

[Bojanowski et al. \(2017\)](#) in particular discusses how adding subword informations, such as trigrams, enriches the encoding of words and leads to reliable compressions. [Svenstrup et al. \(2017\)](#) conduct research on the overloading of different hashfunctions to further improve and compress embedding representations. [Xue and Aletras \(2022\)](#) train BERT-style encoder models based on a different set of hashes on words. [Clark et al. \(2022\)](#) propose a multistage encoding scheme that uses hash functions and convolutions to enhance the BERT-encodings of words.

Another line of work to reduce the vocabulary parameter count is the utilization of weight tying, effectively halving it, as the embedding and head layers become “tied” to the same matrix ([Press and Wolf, 2017](#)). However, the effects on performance are still not sufficiently explored, and it arguably imposes a more difficult training objective.

## 7 Conclusion

In this work we present T-FREE, an alternative to subword tokenizers with a simple and explicitly modeled robust hash function on words. It removes the need and pitfalls to limit “a models potential” to a “pre-pre-trained” vocabulary. We, moreover, fundamentally shift the established target of training language models, previously designed as a single-label problem, into a multi-label prediction based on word similarities. Similarities in particular include leading whitespaces and upper-case variations, for which subword tokenizers add specific tokens that are independently trained from scratch. These contributions allow us to train language models more robust, more adaptable when continuing pre-training with a new language, and with a significantly (to 12.5%) reduced parameter size without a decrease in benchmark scores. Due to the special role of the matrices, the latter in particular allows one to increase micro-batchsize, which further accelerates training time. Finally, the consequent convolution-like encoding achieves SOTA fertility scores across most languages and enables by design synergies to similar language groups. We demonstrated the latter showing that our  $3B$  almost achieved “native-language” performance after a small amount of language-transfer training steps, in contrast to the tokenizer baseline.

## Limitations

With T-FREE we propose a fundamentally different approach to text encoding and decoding in LLMs. Due to the intense resources required to train LLMs, we have focused on evaluating models up to  $3B$  parameters. Evaluations on even larger models and training datasets remain a relevant point of investigation for future work. Nonetheless, we observed an easy transfer from  $1B$  to  $3B$  parameters, and we will continue to train and release more advanced models.

We expect T-FREE to experience some numerical instabilities for very long words since single-word embeddings are calculated as the sum of their  $n \cdot m$  activations. However, less than 2% of the entire slimpajama dataset contains words with more than 10 characters (*cf.* App. I), and we did not encounter any issues with the benchmarks. Consequently, such potential instabilities remain statistically insignificant. Nonetheless, we could adequately tackle long outliers with an additional split rule based on the words length or at the occur-

rence of repetitions. Subword tokenizers already demonstrate that such approaches will work, even when tokens are at first glance meaningless and underutilized—and again, these cases remain outliers. Moreover, a hybrid setup utilizing a large tokenizer (>512k tokens) with T-FREE for optimized memory footprint is depicted in Figure 16.

Similarly, we did not thoroughly study the effect of repetitive trigrams in words. These did also not occur frequently enough to have any measurable effect on our experiments. As of now, we only accumulate a word pattern in a binary fashion, not accounting for trigrams appearing multiple times in a single word. As a fallback, one could again, split words at the position of repetitions. Another promising direction would overload embeddings with positional encodings similar to rotary (Su et al., 2024).

Although T-FREE’s fertility on code is on par with that of LLama2 (cf. App. E), it could be further improved by explicitly modeling code patterns. In this work, we have focused on natural language and leave detailed evaluations of T-FREE in downstream coding tasks for future research. Furthermore, we did not investigate languages entirely relying on Unicode byte-encodings, such as Chinese. However, as they seemingly work out-of-the-box with subword tokenizers, we do not expect issues here by splitting them character/ word-wise. In particular for asian symbols, additionally translating the symbols to its romanization through the phonetic alphabet such as pinyin may further improve the synergies of word encodings.

Finally, we only studied a single constructed hash function for T-FREE. As this work paves the way to model required language features more explicitly, we are looking forward to variations of the proposed T-FREE method.

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## Appendix

### A T-FREE Algorithm

Alg. 1,2,3,4,5 show the core steps to encode text into embeddings, and decode text from model predictions with T-FREE. Here, *regex.split* denotes an algorithm that splits text based on a regular expression, *hash* denotes an arbitrary hash function like *md5*,  $\%$  denotes the mathematical modulo operation. In style of python,  $f'\{token\}_-$  denotes text formatting to indicate the string with content of variable *token* being followed by an underscore, and  $EL[i]$  denotes the  $i$ -th entry of matrix *EL* and  $'string'[i : i + 3]$  three consecutive characters in the text *string* starting from position  $i$ , where  $'s'$  is at position 0. Finally,  $v \approx 8,000$  is the chosen vocabulary size,  $d \approx 100,000$  is the chosen dictionary size,  $h \approx 3,072$  the LLMs hidden size. Finally,  $\mathbb{0}^h$  denotes a zero vector of dimension  $h$  and  $\mathbb{1}^{v \times d}$  a matrix with entries 0 or 1. Note that we included some normalization steps in Alg. 5, which we surprisingly found not beneficial for Alg. 3 in our ablations.

Finally, refer to Figure. 14,15 for a step-wise comparison of the computation step, parameters and runtimes. Figure 16 shows a “hybrid” mode, in which embody a classical subword-tokenizer as a text preprocessing step, but utilize T-FREE to keep the “tokenizer free LLM backbone”. Arguably, this approach benefits from a compressed embedding layer, and the tokenizer may easier be exchanged afterwards—the encoding of the text-chunks in the backbone will be kept as proposed.

---

#### Algorithm 1 token\_split

---

**input:** *text*  
 $tokens \leftarrow \text{regex.split}((\_|W|d), text)$   
*(cf. Sec. B if necessary)*  
**output:** *tokens*

---

### B Whitespace encoding

By default our model is trained to predict full words separated by whitespaces. To not be limited to this use-case, we add a special “non-whitespace” and “whitespace” token. We empirically evaluated each exception occurring in code tokenization. To further reduce its fertility, we favor “non-whitespace” before one of the following characters:

$\$. , ; : \# ? ! = - + * / \backslash ( ) < > [ ] \& \% _ \sim ^$

---

#### Algorithm 2 trigramify

---

**input:** *token, k, m*  
 $\triangleright k$  : lowercase activation,  $m$  : total activation  
 $pattern \leftarrow \mathbb{0}^v$   
**for**  $l \in [0, \text{len}(token) - 1]$  **do**  
 $trigram \leftarrow f'\{token\}_-[l : l + 3]$   
**for**  $i \in [1, m]$  **do**  
**if**  $i \leq k$  **then**  
 $string_i = \text{lower}(trigram)$   
**else**  
 $string_i = trigram$   
**end if**  
 $hash_i = \text{hash}(f'\{string_i\}_{-}\{i\}')$   
 $pattern[hash_i \% v] = 1$   
**end for**  
**end for**  
**output:** *pattern*

---



---

#### Algorithm 3 encode

---

**input:** *token, EL*  
 $\triangleright EL$ : Embedding Layer ( $\in \mathbb{R}^{v \times h}$ )  
 $embedding \leftarrow \mathbb{0}^h$   
 $pattern \leftarrow \text{trigramify}(token)$   
**for**  $i \in [0, v - 1]$  **do**  
**if**  $pattern[i] == 1$  **then**  
 $embedding \leftarrow embedding + EL[i]$   
**end if**  
**end for**  
**output:** *embedding*

---

We further prefer non-whitespace after one of the following characters:

$\#\$ = - + * / ' \backslash " ( < [ \sim ^ \& \% _ \backslash n 1 2 3 4 5 6 7 8 9 0$

As such, the text “In 2024” would result in the split “[In,2,0,2,4]” without the need of any special annotations, while “In20 24” resolves to “[In,<no\_ws>,2,0,<ws>,2,4]”.

Finally, to further improve code fertility, we merge consecutive <ws> and newline tokens up to 3 times, i.e. 8 consecutive whitespaces would result in a single <l8<ws>> token.

### C Tokenizer trainings with sentencepiece

For training of a unigram tokenizer with the current sentencepiece library, a 20GB reference data corpus reaches the limit of our available 1TB Ram compute node. We thus randomly sample 20GB of the slimpajama dataset and run the following

**Algorithm 4** compile\_dictionary

---

**input:** *tokens*  $\triangleright d$  target tokens  
 $dict \leftarrow \mathbb{O}^{d \times v}$   
**for**  $i \in [0, d - 1]$  **do**  
     $dict[i] \leftarrow \text{trigramify}(tokens[i])$   
**end for**  
**output:** *dict*

---

**Algorithm 5** decode

---

**input:** *logit*, *dict*, *tokens*  
 $\triangleright$  *logit*: single prediction ( $\in \mathbb{R}^{v \times 1}$ ),  
*dict*: compiled dictionary ( $\in \mathbb{1}^{d \times v}$ ),  
*tokens*:  $d$  tokens corresponding to *dict*  
 $scores \leftarrow dict \cdot \text{sigmoid}(logit)$   
**for**  $i \in [0, d - 1]$  **do**  
     $scores[i] \leftarrow scores[i] / \text{sum}(dict[i])$   
**end for**  
 $scores \leftarrow \text{softmax}(scores)$   
 $i \leftarrow \arg \max_l scores[l]$   
**output:**  $tokens[i]$ ,  $scores[i]$

---

statement for training of the actual tokenizer:

```
spm_train --input=20GB_sample.txt \
--model_prefix=unigram_64k \
--vocab_size=64000 \
--character_coverage=0.99 \
--model_type=unigram \
--byte_fallback=true \
--split_by_number=true \
--split_by_whitespace=true \
--train_extremely_large_corpus=true \
--split_digits=true \
--allow_whitespace_only_pieces=true \
--remove_extra_whitespace=false \
--normalization_rule_name=nfkc \
--num_threads 64 --eos_id=0 \
--bos_id=-1 --unk_id=2 \
--pad_id=1 \
--eos_piece="<|endoftext|>" \
--pad_piece="<|padding|>" \
--unk_piece="<|unknown|>"
```

**D Training Configurations****D.1 1B**

Training Parameters are listed in Tab. 2.

**D.2 3B**

Training Parameters are listed in Tab. 3.

Parameter	Value
hidden size	2,048
layers	16
attention heads	16
norm	layer
mlp	gelu
mlp scale	5,456
training steps	50k
sequence length	2,048
batch size	1,024
precision	bfloat16
learning rate	6e-4
minimum learning rate	6e-5
annealing	cosine
annealing steps	50k
warmup steps	200
optimizer	AdamW
optimizer beta1/ beta2/ eps	0.9 / 0.95 / 1e-8
weight decay	0.1

Table 2: 1B Parameter configurations (for all ablations).

Parameter	Value
hidden size	3,072
layers	24
attention heads	24
norm	rms
mlp	swilu
mlp scale	8,192
training steps	90k (20k)
sequence length	4,096
batch size	1,024
precision	bfloat16
learning rate	3e-4 (1e-4)
minimum learning rate	3e-5 (3e-5)
annealing	cosine
annealing steps	90k (20k)
warmup steps	200 (500)
optimizer	AdamW
optimizer beta1/ beta2/ eps	0.9 / 0.95 / 1e-8
weight decay	0.1

Table 3: 3B Parameter configurations (for all ablations). In brackets are highlighted values for German continued pre-training.

## E Fertility Analysis

We subsequently provide further experimental details on the fertility analysis conducted with respect to **F3**, Sec. 4.4. As a reference dataset, we used the November 23 dump of Wikipedia in the respective languages. We derived reference tokenization using UDPipe (Straka, 2018). A tokenizer’s fertility is then calculated by dividing its total token count for a document by the number of tokens produced by UDPipe. We present results for more models on 8 languages in Tab. 5.

We also evaluated the white-space tokenization of T-FREE for code. For 22 programming languages, we took 10k random documents each from the starcoder dataset<sup>14</sup>. Since ground truth text splitting for code is hard to establish, we instead report the normalized sequence length with respect to a reference tokenizer. We here used Llama-2 and report results in Tab. 4. Since T-FREE’s tokenization achieves an NSL close to 1.0, it performs roughly on par with Llama-2.

## F Token Overlap/Duplicates

For the empirical evaluation regarding **F2**, cf. Sec. 4.3, we present more exhaustive results with additional models in Tab. 6.

## G Training stability

Memory footage comparing classic tokenizers to T-FREE is found in Fig. 6.

Note that the hashing step of Alg. 2 uniformly distributes gradients amongst the available vocabulary, as discussed in Sec. 5. This is in contrast to classic tokenizers, as they depend on a bijective single-label mapping, and as such each vocabulary entry update is dependent on its the occurrence frequency of the corresponding token within the dataset. Moreover, we explicitly let trigram activations overlap with their lowercase version. We assume that these are responsible for the more stable training dynamics as shown in Fig. 5. Moreover, we found that the lowercase overlap bootstraps learning as shown with the downstream benchmark ablations Fig. 8.

## H Hyperparameter Ablations

*Some 1,500 determined experiments later...*

<sup>14</sup><https://huggingface.co/datasets/bigcode/starcoderdata>

lang	Ours (NSL) ↓	Starcoder (NSL) ↓
c-sharp	1.034783	0.816206
c	0.996308	0.860453
cpp	1.084867	0.855094
css	1.109492	0.903693
cuda	1.018222	0.857034
dockerfile	0.954086	0.851568
go	1.142476	0.883456
html	1.164936	0.885237
java	1.003201	0.835858
javascript	1.183923	0.850398
json	1.071685	0.892871
kotlin	0.925868	0.846053
makefile	1.006108	0.862994
markdown	0.965325	0.892784
php	1.179374	0.838566
python	1.005064	0.857439
ruby	0.979135	0.846597
rust	1.086027	0.857645
shell	1.041322	0.879112
sql	0.954786	0.859785
typescript	1.121119	0.847393
yaml	0.974146	0.856218
<b>Overall</b>	1.045557	0.860748

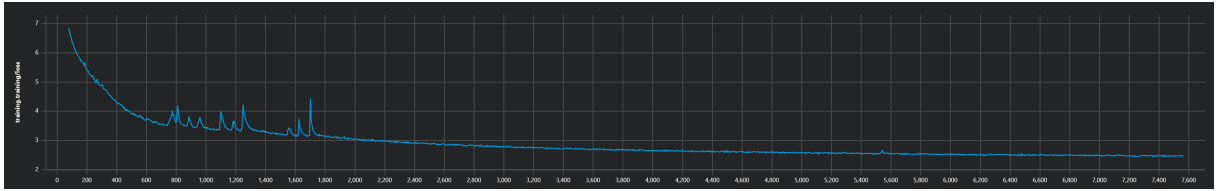
Table 4: Normalized sequence length wrt Llama-2 on code tokenization.

Model	EN	DE	FR	ES	IT	RU	VI	AR
Unigram Baseline (32k)	1.3584	2.2577	2.1354	2.1524	1.9508	11.4448	5.1826	9.4740
Unigram Baseline (64k)	1.2802	2.0043	1.9492	1.9163	1.7263	11.4305	5.0603	9.4555
BPE Baseline (32k)	1.3585	2.2784	2.0625	2.0977	1.9396	11.4321	4.8717	9.4694
BPE Baseline (64k)	1.2759	2.0253	1.9059	1.8894	1.7212	11.4231	4.7545	9.4656
Mistral (32k)	1.3973	1.9316	1.6613	1.7569	1.7591	2.5601	3.3458	4.7228
Llama-2 (32k)	1.4014	1.7702	1.5495	1.6413	1.6160	2.3242	3.3540	4.8255
Phi-2: (50k)	1.2654	2.2660	1.8183	1.9736	1.9132	6.7289	4.3392	5.2246
Gemma (256k)	1.1761	1.4470	1.2754	1.3163	1.3253	1.9028	1.7257	1.7938
DBRX (100k)	1.2381	1.8311	1.5423	1.6142	1.6191	3.2385	2.6617	3.6821
Jais (85k)	1.3029	2.1391	1.7347	1.8514	1.8244	3.6730	3.4382	1.2653
Command-R (255k)	<b>1.1525</b> •	1.4110	1.2079	1.2527	1.2460	1.5899	1.5967	1.5787
Llama-3 (128k)	1.2330	1.8221	1.5351	1.6033	1.6130	2.2144	1.8261	1.9660
NeMo-Tekken (131k)	1.2313	1.5178	1.3061	1.3845	1.4171	2.0521	1.8378	1.6045
<b>Ours</b>	<b>1.1636</b> ◦	<b>1.1829</b>	<b>1.2363</b>	<b>1.1695</b>	<b>1.1274</b>	<b>1.3386</b>	<b>1.4001</b>	<b>1.0863</b>

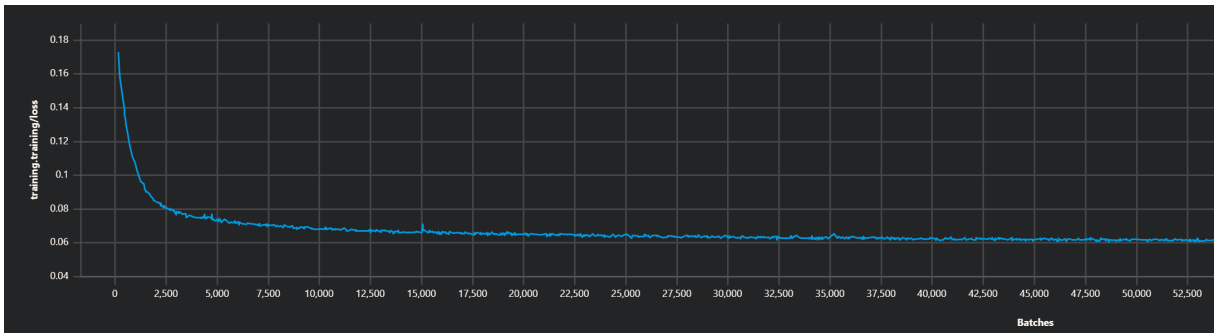
Table 5: Additional evaluations of fertility evaluations. *Cf.* Sec. 4.3.

Model/Tokenizer	Portion of duplicate tokens (%) ↓			
	Total	Cap.	Space	Digits
Unigram Baseline (32k)	32.99	21.44	11.76	0.00
Unigram Baseline (64k)	35.24	23.27	13.47	0.00
BPE Baseline (32k)	32.12	21.30	13.85	0.00
BPE Baseline (64k)	35.32	23.82	15.52	0.00
Phi-2: (50k)	23.23	12.91	16.89	3.32
DBRX (100k)	24.87	23.77	16.17	1.10
GPT-2 (50k)	25.25	21.93	16.99	3.32
Gemma (256k)	34.68	20.27	20.50	0.04
Command-R (255k)	15.31	15.31	14.00	0.00
Mistral (32k)	31.47	19.10	16.45	0.00
Llama-2 (32k)	30.23	17.10	16.98	0.00
Llama-3 (128k)	21.22	20.17	15.28	1.05
NeMo-Tekken (131k)	23.12	13.30	11.99	0.00
<b>T-Free (Ours)</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>

Table 6: Additional evaluations of overlap of full tokens occurring multiple times, only with capitalization or whitespace in difference. Note that there are still plenty more redundancies with sub-token reconstructions. *Cf.* Sec. 4.3.



(a) Classic Tokenizer.



(b) T-FREE.

Figure 5: Exemplary comparison of classic tokenizer ( $v = 64k$ ) training loss curve (top) and T-FREE ( $v = 16k$ ) training loss (bottom). Overall we noticed less spikey training behavior when using T-FREE. Both 3B models were trained on same slimpajama data, token-batchsize and learning rate  $4.5e-4$ .



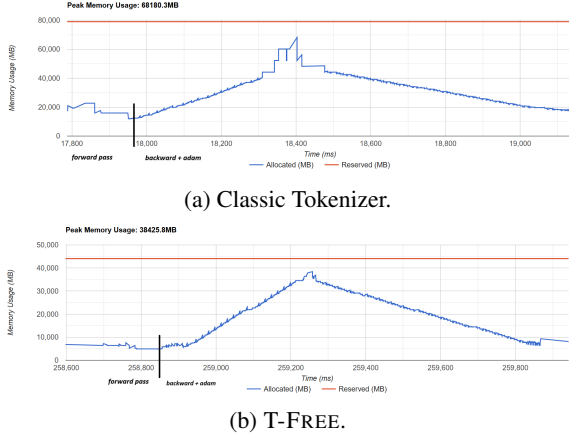


Figure 6: Pytorch Profiler Memory Footprint of a single forward and backward pass on a 1B, each with batch size 8 and 4k sequence length. Top is classical tokenizer version with 64k vocab size, bottom trigram with 8k vocabulary. Note how AdamW aggregates peak memory consumption until 68GB for classic tokenizer, while ours remains at 38GB.

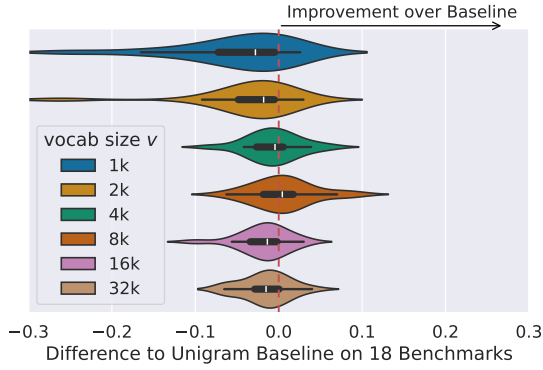


Figure 7: Further ablations on hyper parameters in accordance to Fig. 3. Note that after peaking at  $v = 8k$ , performance slightly decreases again, which may be attributed to the undertrained stage of the model trainings.

Albeit pretty scarce, some more hyper-parameter ablations are found in Fig. 7,8.

We will continue to polish and add more...

## I Some Statistics

**Trigram combinatorics.** As there are more than  $v$  possible words, there will naturally be some overlap in the activations between words. However, assuming an embedding dimension of  $v \approx 8,000$ ,  $m \approx 8$  activations per trigram, and a word of length  $n = 5$ , there are (in theory)  $\binom{v}{n \cdot m} \approx 10^{108}$  unique activation patterns.

This overlap can be interpreted as an interpolation between input states. For entirely independent inputs, this overlap should be kept small as the re-

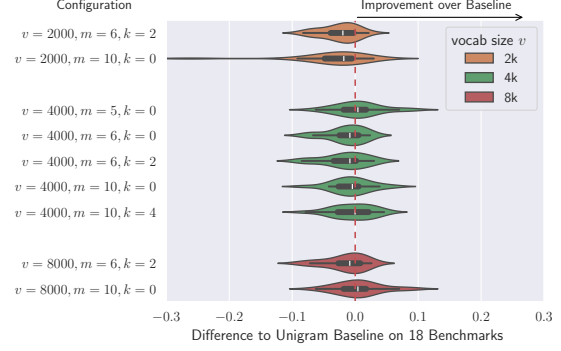


Figure 8: Further ablations on hyper parameters.

sults cannot benefit from the states of the shared activations. As such, we require a *robust hash function* on text, i.e. a mapping from text into sparse activation patterns, for which the overlapping of activations is proportional to the similarity of the input words. We model this through trigrams, and as such, letter-similarity.

**Tokenizer Duplicates.** Tab. 7 shows the curse of token-based vocabularies: to produce all 64 upper and whitespace variations of the word “\_words”, one requires on average 3 tokens per writing.

**Dataset-Coverages.** Fig. 9 shows the covered percentages of the entire dataset, by word-lengths, for all slimpajama datasets. If we successfully can encode all words of length  $\leq 10$ , we can cover  $\geq 95\%$  of the entire slimpajama dataset. Or conversely, we would only require 5% outlier handling/additional splits for longer words (cf. Sec. 7).

Fig. 10 and Fig. 11 show dataset coverage (y-axis) of top-n words and trigrams (x-axis) for each slimpajama category. Notably 10k trigrams, and 100k words consistently cover  $> 95\%$  of each slimpajama category.

## J More Benchmarks

We used the code of the eleuther eval harness, and evaluated each benchmark in 0-shot and 2-shot. All 18 benchmarks, namely *arc (easy and challenge)*, *hellaswag*, *winogrande*, *triviaqa*, *xnli*, *truthfulqa*, *boolq*, *copa*, *openbook*, *pqa*, *multirc*, *lambda (openai and standard)*, *race*, *rte*, *wic*, *webqs* are visualized in Fig. 12 and Fig. 13 for a baseline model trained on english slimpajama only and continued finetuning on german occiglot. Arc-challenge, hellaswag, xnli and truthfulqa are also evaluated in german translations. Detailed numbers can be found in Tab. 8,9 and 10.

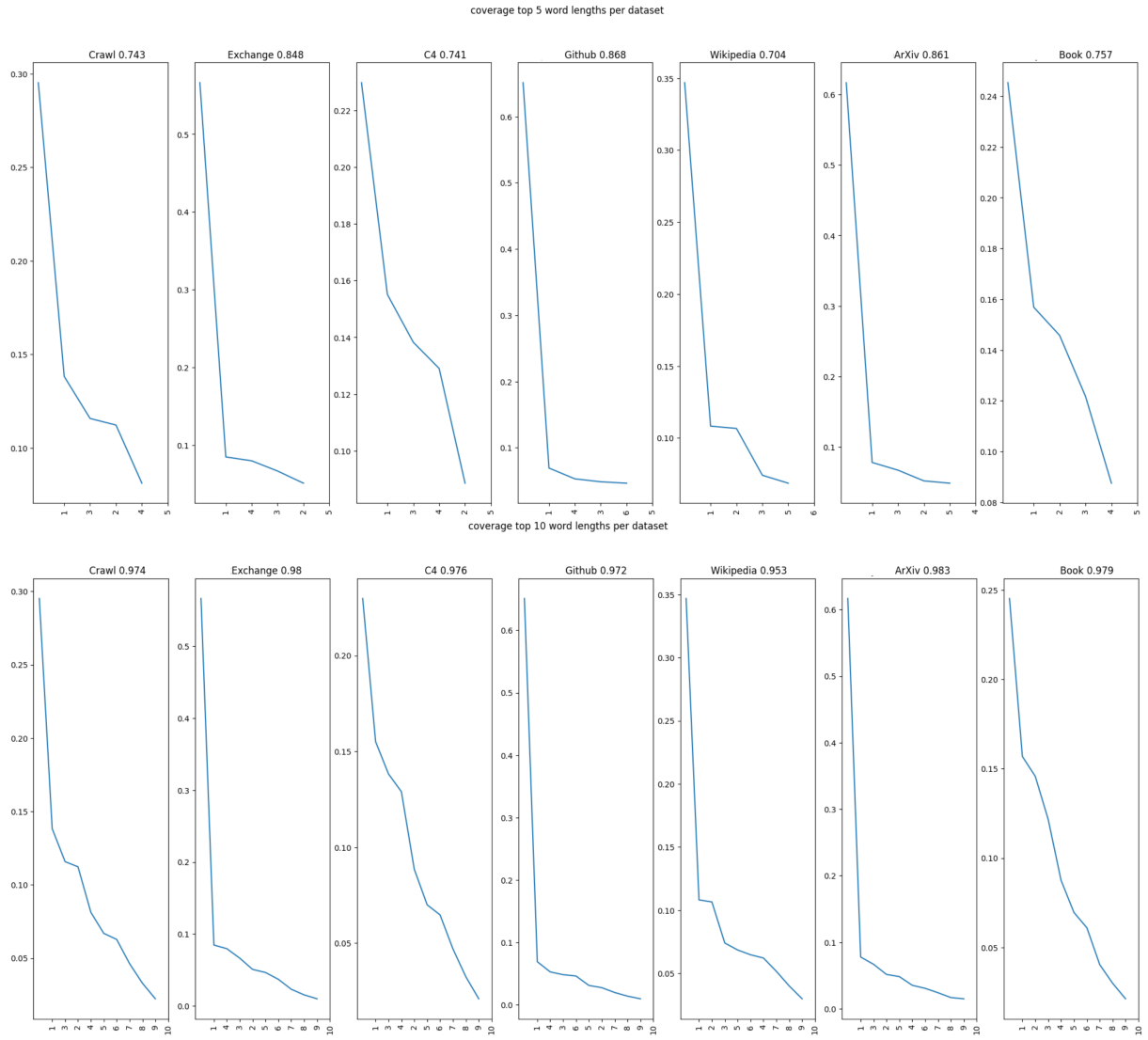


Figure 9: Top 5 and top 10 occurring word lengths (x-axis) per slimpajama data category, with coverage-percentage (y-axis). Headline indicates total percentage covered by top  $n$ -length words. With words of length  $\leq 5$ , one always covers  $\geq 74\%$  of all occurring words. With all words of length  $\leq 10$ , one achieves  $\geq 95\%$ .

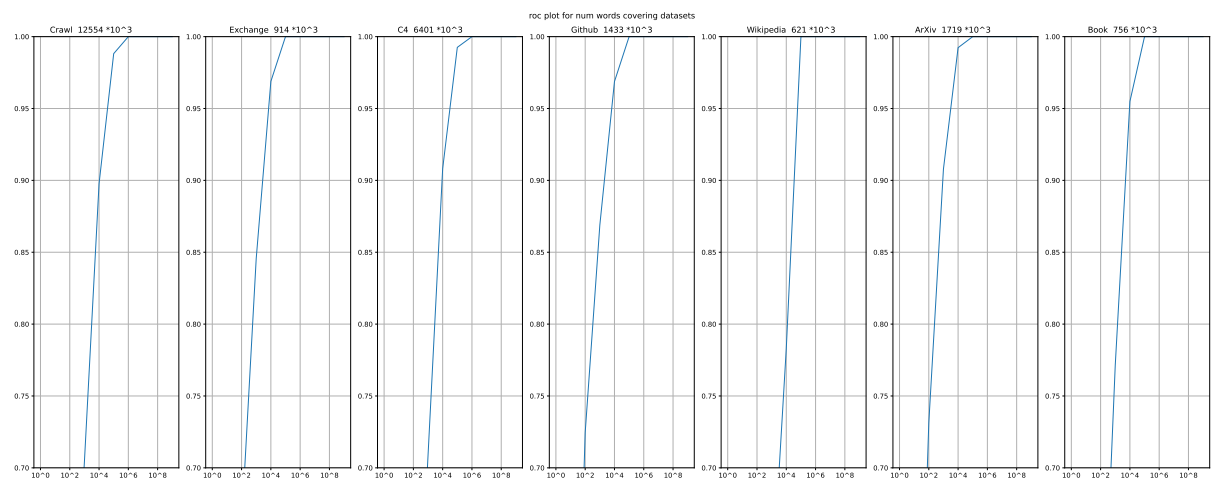


Figure 10: Most-frequent word coverage of Slimpajama categories. Title shows the total number of words per dataset sample,  $x$ -axis the top- $n$  chosen words,  $y$ -axis the percentage covered within dataset. With only 100k words we can consistently cover  $> 95\%$  of each category.

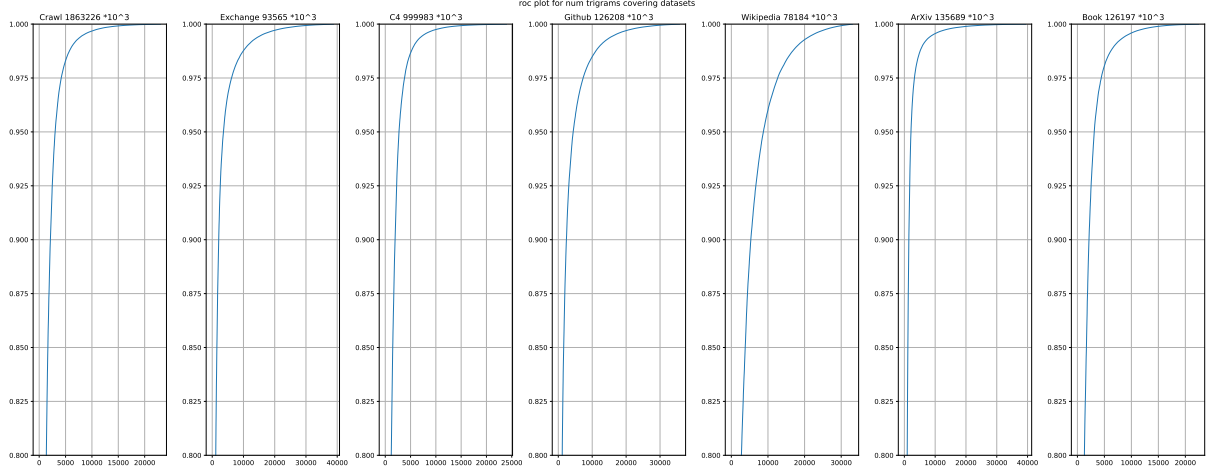


Figure 11: Number of trigrams ( $x$ -axis) required to cover ( $y$ -axis) percentage of the categories of slimpajama (total number words sampled in title). With only 10k trigrams we can cover  $> 95\%$  of all occurring words.

string	token id
‘ ’	49594
‘_w’	15997
‘W’	40669
‘_W’	46854
‘w’	63048
‘Wo’	7411
‘_wo’	14297
‘_WO’	14883
‘wo’	34034
‘_Wo’	39790
‘WO’	44468
‘WOR’	1916
‘_WOR’	6606
‘_Wor’	40813
‘_Word’	1971
‘Word’	3212
‘_word’	14272
‘WORD’	48022
‘word’	49922
‘_words’	12555
‘words’	28689
‘WORDS’	32751
‘_Words’	37912
‘Words’	51858

Table 7: The 24 possible first tokens to construct uppercase and whitespace variations of “\_words”, where “\_” denotes a whitespace. In total, there are 64 ways to write “\_words”, which requires  $32 \cdot 6 + 32 \cdot 5 = 342$  characters. The tokenizer requires in total 194 tokens, of which 37 are unique, leading to an average (neglecting the occurrence frequencies) of  $\approx 3$  tokens per writing.

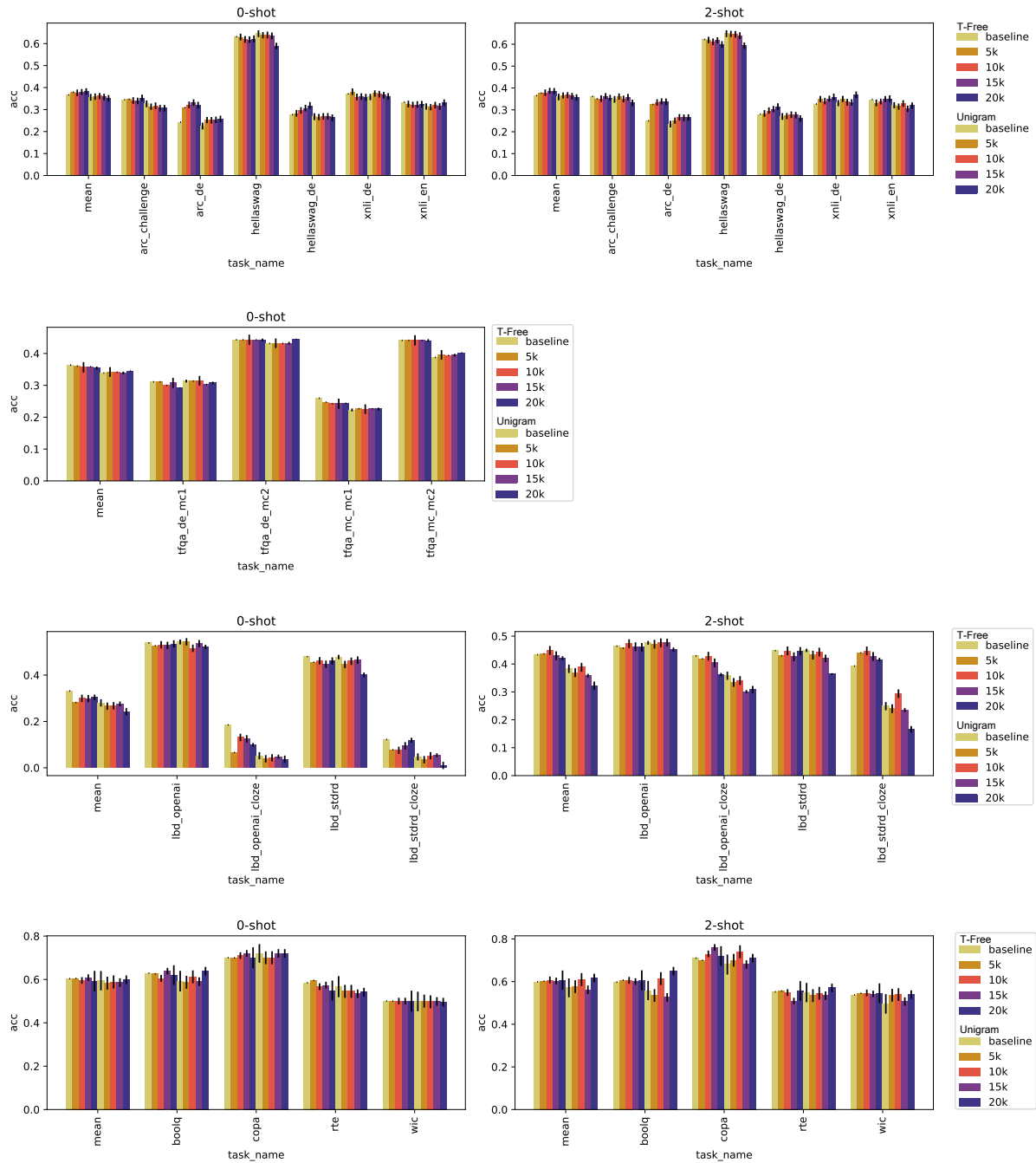


Figure 12: Detailed benchmark results on evaluations of Sec. 4.4.



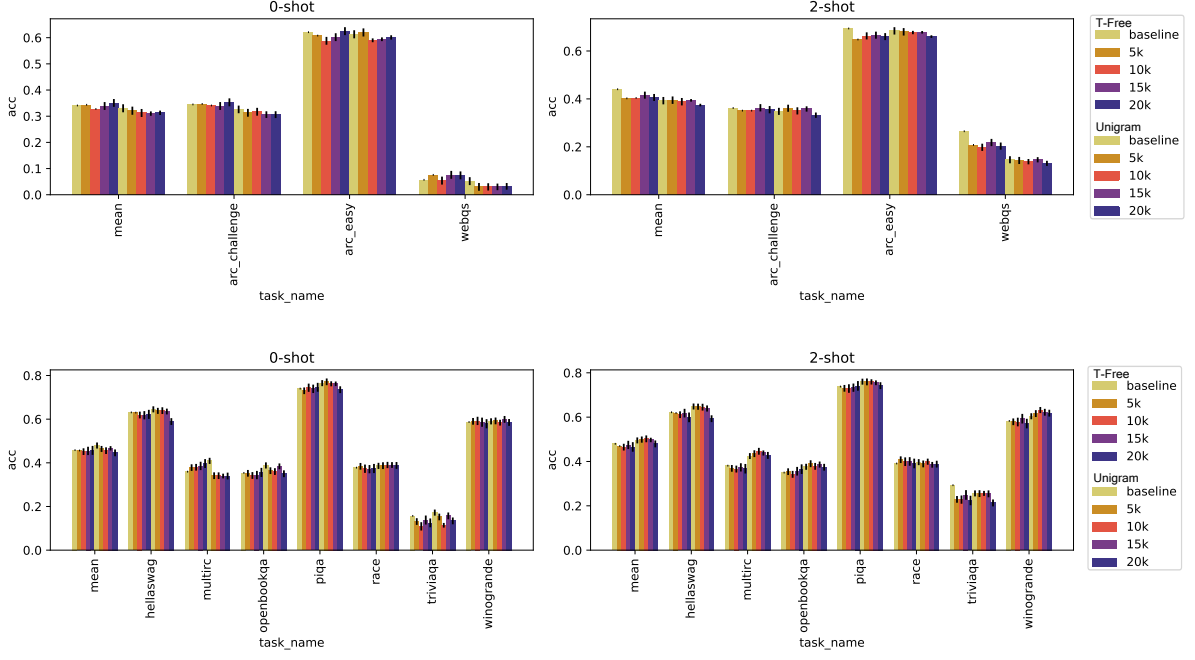


Figure 13: Further detailed benchmark results on evaluations of Sec. 4.4.

Model		english benchmarks					german benchmarks				
		<i>arc<sub>ch</sub></i>	<i>hella</i>	<i>xnli</i>	<i>tf<sub>mc1</sub></i>	<i>tf<sub>mc2</sub></i>	<i>arc<sub>ch</sub></i>	<i>hella</i>	<i>xnli</i>	<i>tf<sub>mc1</sub></i>	<i>tf<sub>mc2</sub></i>
base	T-Free	34.5/36.2	63.1/62.2	33.4/34.8	25.9/-	44.1/-	24.3/25.0	27.7/28.1	37.1/32.7	31.1/-	44.3/-
	Unigram	32.6/34.8	64.5/64.8	31.4/32.1	22.3/-	38.8/-	22.5/23.5	26.8/27.0	35.7/33.1	31.3/-	43.1/-
5k	T-Free	34.7/35.1	63.0/61.9	32.5/33.2	24.7/-	44.1/-	30.9/32.5	28.3/28.4	38.1/34.9	31.1/-	44.3/-
	Unigram	31.3/36.1	63.8/64.7	31.1/31.5	22.6/-	39.6/-	25.2/25.1	26.6/27.2	37.3/35.0	31.3/-	43.2/-
10k	T-Free	34.1/35.1	61.9/61.1	32.2/33.8	24.2/-	44.1/-	32.1/33.4	29.6/29.7	35.7/34.0	30.0/-	44.3/-
	Unigram	31.7/35.0	63.9/64.5	32.0/32.9	22.5/-	39.4/-	25.1/26.6	26.9/27.8	37.0/33.5	31.5/-	43.1/-
15k	T-Free	33.9/36.3	61.8/61.8	32.3/34.9	24.2/-	44.1/-	33.2/33.9	30.7/30.4	35.9/35.1	30.8/-	44.3/-
	Unigram	30.6/35.8	63.5/63.9	31.4/30.5	22.6/-	39.4/-	25.3/26.4	26.9/27.6	36.5/33.4	30.2/-	43.2/-
20k	T-Free	35.3/35.5	62.2/59.9	32.4/35.1	24.4/-	44.1/-	32.0/33.7	31.9/31.5	35.6/35.9	29.1/-	44.2/-
	Unigram	30.6/33.2	58.9/59.3	33.1/31.9	22.6/-	40.2/-	25.7/26.5	26.3/26.1	36.0/36.9	30.7/-	44.4/-

Table 8: Accuracy scores of english and german translated benchmarks for continued pre-training. First value denotes 0-shot, second value 2-shot (if available). Notably, the T-Free baseline model slightly outperforms (or performs on par with) the Unigram baseline model on all of these tasks. On german evals of arc and hellaswag, the T-Free baseline outperforms Unigram, and achieves larger gains during continued training on the german/ english data mix. The german versions of xnli and truthfulqa mostly remain unchanged.

Model		english benchmarks							
		<i>arc<sub>ez</sub></i>	<i>boolq</i>	<i>copa</i>	<i>wino</i>	<i>obook</i>	<i>piqa</i>	<i>trivia</i>	<i>mrc</i>
base	T-Free	62.1/69.4	62.9/59.8	70.0/71.0	58.7/58.3	35.4/35.0	74.0/73.8	15.7/29.3	36.0/38.1
	Unigram	61.3/68.4	59.2/55.8	72.0/68.0	59.0/60.4	38.8/37.6	76.5/76.0	17.2/25.6	40.9/42.4
5k	T-Free	60.8/64.8	62.6/60.7	70.0/70.0	59.0/57.9	35.2/35.4	73.0/73.0	13.1/22.9	37.8/36.9
	Unigram	62.0/68.0	58.7/53.5	70.0/70.0	59.2/61.6	36.4/39.0	77.1/75.9	15.2/25.5	34.1/43.5
10k	T-Free	58.8/66.3	60.5/60.6	71.0/73.0	59.2/57.8	34.2/34.2	74.4/72.9	10.9/22.8	38.0/36.6
	Unigram	59.0/67.7	61.2/61.4	70.0/74.0	58.4/63.2	36.0/37.8	76.2/75.9	11.4/25.6	34.1/44.6
15k	T-Free	60.2/66.6	63.8/60.0	72.0/76.0	58.6/59.3	34.4/35.6	73.9/73.3	13.8/24.9	38.6/37.3
	Unigram	59.4/67.8	59.0/52.6	72.0/68.0	59.9/62.2	38.4/38.6	76.2/75.5	15.7/25.4	34.0/43.8
20k	T-Free	62.5/66.0	62.0/62.6	70.0/72.0	57.9/57.1	35.6/36.5	74.6/74.1	12.5/22.4	39.7/36.7
	Unigram	60.1/66.0	63.8/62.9	72.0/71.0	58.5/61.8	35.0/37.4	73.5/74.2	13.4/21.3	34.0/42.7

Table 9: Accuracy scores of english benchmarks for continued pre-training. First value denotes 0-shot, second value 2-shot. Notably, the T-Free model performs on par to the Unigram model on all of these tasks, throughout the entire continued training.

Model		english benchmarks							
		$lbd^{oai}$	$lbd_{clz}^{oai}$	$lbd^{stdr}$	$lbd_{clz}^{stdr}$	<i>race</i>	<i>rte</i>	<i>wic</i>	<i>webqs</i>
base	T-Free	53.9/46.4	18.5/42.9	48.0/44.8	12.2/39.3	37.8/39.1	58.5/55.2	50.0/53.8	5.7/26.5
	Unigram	54.3/47.6	5.2/35.8	47.8/44.9	4.7/24.9	38.6/39.7	56.7/54.9	49.8/49.5	5.2/14.6
5k	T-Free	52.7/45.7	6.6/41.9	45.6/43.0	7.8/44.0	38.4/40.8	59.6/55.6	50.0/54.5	7.6/20.8
	Unigram	54.4/47.1	3.9/33.4	44.6/43.2	3.5/24.0	38.7/38.9	54.5/53.4	50.0/53.6	3.0/14.3
10k	T-Free	53.1/47.3	13.1/42.8	46.2/44.7	7.5/44.7	37.3/39.9	56.7/54.9	50.0/54.7	5.4/19.8
	Unigram	51.6/47.6	4.3/34.1	46.0/44.3	5.2/29.4	39.0/39.9	54.5/54.5	49.7/53.9	3.0/13.8
15k	T-Free	52.8/46.2	12.5/40.4	44.7/42.6	9.5/42.7	37.1/40.0	57.4/50.9	50.0/54.2	7.6/21.8
	Unigram	53.6/47.7	4.8/30.1	46.5/42.0	5.4/23.5	39.0/38.5	53.4/53.4	49.8/50.6	3.0/14.6
20k	T-Free	53.5/46.0	9.9/36.2	46.1/44.6	11.9/41.6	37.4/39.1	54.9/55.6	50.0/54.5	7.4/20.3
	Unigram	52.2/45.2	3.7/30.8	40.1/36.4	1.0/16.6	38.8/38.8	54.2/57.0	49.5/53.9	3.2/13.1

Table 10: Accuracy scores of english benchmarks for continued pre-training. First value denotes 0-shot, second value 2-shot. The T-Free model performs on par with the Unigram model on all of these tasks, throughout the entire continued training. Notably, the clozed variants of lambda are most fragile, at which T-Free outperforms.

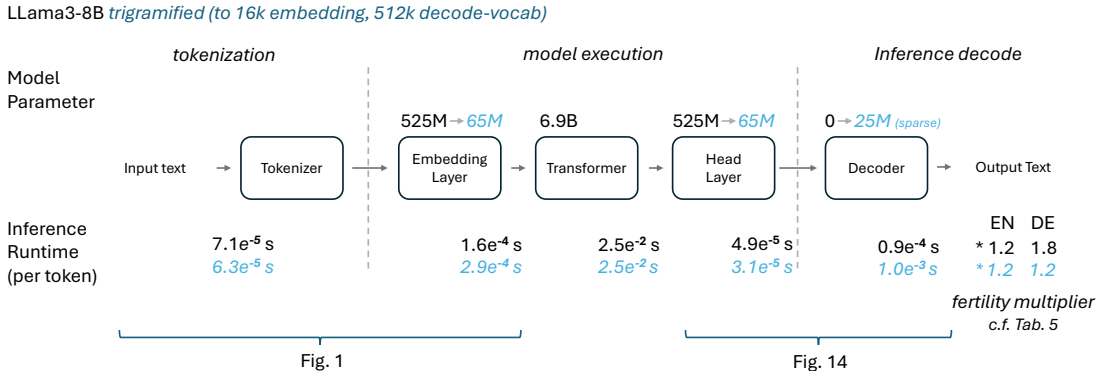


Figure 14: Comparison of the end-to-end LLM processing steps for the standard LLama3-8B model versus a proposed *trigramified* version. In particular, the two biggest matrices of the model, the embedding layer and the head can be significantly compressed, which can half the training resources when using standard libraries (c.f. Fig. 6). Otherwise, the training execution time is mostly on par. For decoding, the proposed T-Free version requires an additional step to predict the next word. We assumed a vocabulary of 512k entries with an average of 50 activations per entry. This leads to additional 25M nonzero parameters that can be casted into a sparse matrix format (c.f. Fig. 15). The overall inference run-time increases slightly when averaging the entire pipeline processing time, but the biggest consumption remains at the actual transformer backbone. However, note that in addition training and inference time benefit from the improved fertility factors of T-Free. Furthermore, depending on the use case, smaller dictionaries, faster sparse hardware accelerators, or different decoding strategies may be applicable.

### LLama3-8B *trigramified* (16k vocab)

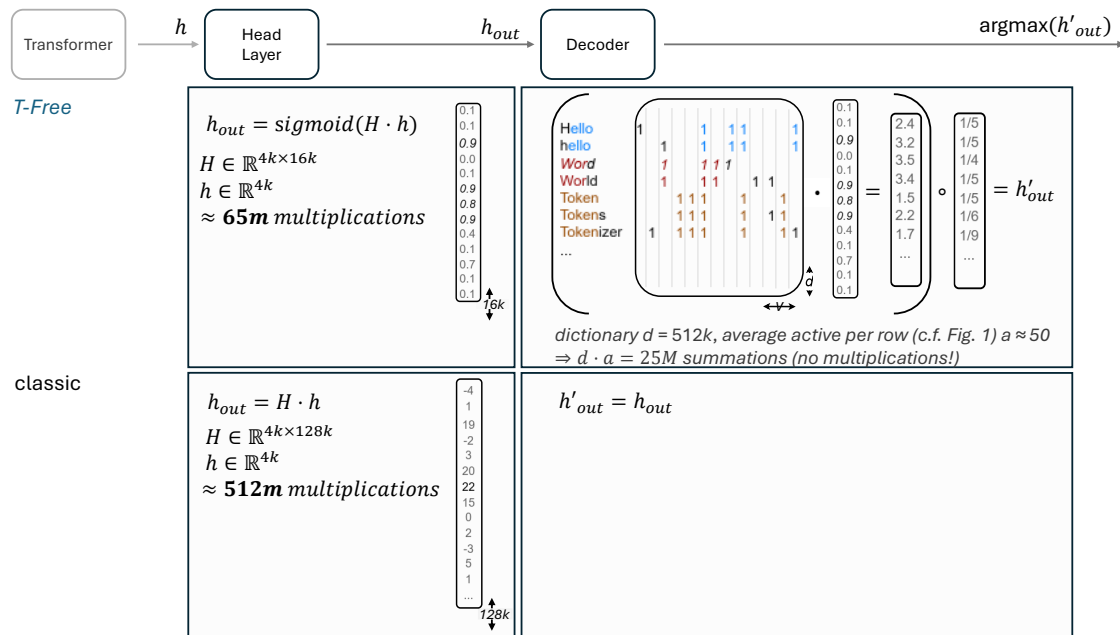


Figure 15: “Greedy” text-decoding example for T-Free (top) and classic decoder LLMs (bottom). T-Free applies a head of significantly reduced parameters which results in less dense matrix multiplications and smaller vector sizes. As an additional step, during inference, T-Free computes the *average activation score*  $h'_{out}$ , which is sparsely computed by multiplying (and averaging) the once precomputed decodable dictionary with the sigmoid scores of the head. Finally, in both cases argmax is taken to lookup the resulting word.

### Potential 1: Tokenizer-Free backbone, large (exchangeable) tokenizer *trigramified hybrid LLM backbone/ example LLama3 8B*

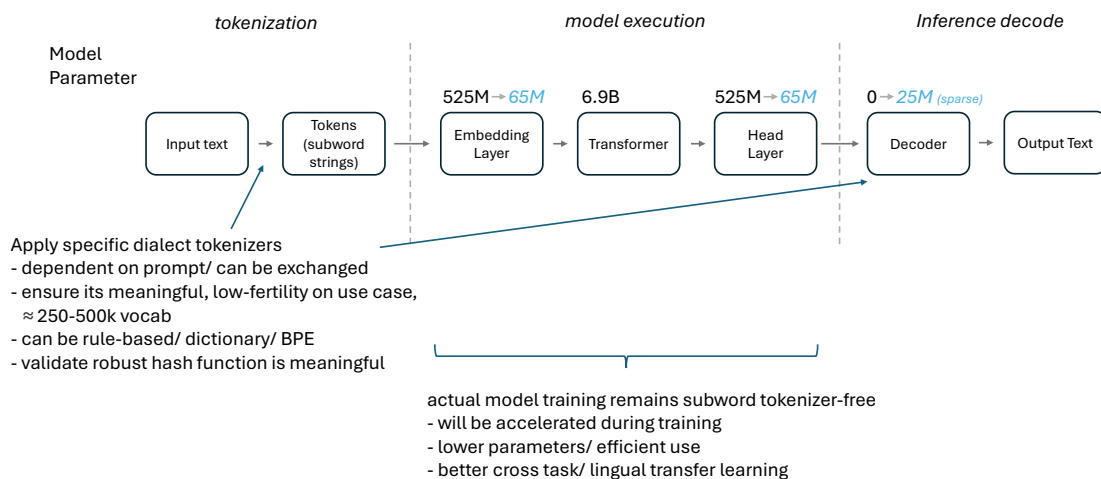


Figure 16: Hybrid “T-Free” (tokenizer-free/adaptable) LLM Backbone applying large scale (500k+) tokenizers. Major advantages of a T-Free backbone in a hybrid setting are the compression of embedding and head matrices, and the potential flexibility to lateron exchange (with some finetuning) the tokenizer — the backbone remains with the same tokenizer-free encoding rules.