Parsing of Context-Free Grammars

Bernd Kiefer

{Bernd.Kiefer}@dfki.de

Deutsches Forschungszentrum für künstliche Intelligenz

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Limits of Regular Expressions

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  - ➢ Regular expressions can only count *finite* amounts of brackets
  - ➢ We need a more powerful formal device: *context-free grammars*

Limits of Regular Expressions

- Assignment: Write a regular expression for fully bracketed arithmetic expressions!
- Answer:
  - ➢ This is not possible!
  - ➢ Regular expressions can only count *finite* amounts of brackets
  - ➢ We need a more powerful formal device: *context-free grammars*
  - ➢ Context-free grammars provide a (finite) inventory of named brackets
  - ➢ All regular languages are also context-free, i.e.: for every regular expression, there is a context-free grammar that accepts / derives the same language

A context-free grammar (CFG) consists of:

- The set of terminal symbols $\Sigma = a, b, c, \ldots$ (the words or letters of the language)
- The set of non-terminal symbols $N = A, B, C, \ldots$
- The startsymbol $S \in N$
- The set of productions (rules) P, where $P \ni r = A \rightarrow \alpha$ with $\alpha \in (\Sigma \cup N)^*$ (we use greek letters for strings of $\Sigma \cup N$)

Example: A grammar for arithmetic expressions:

$$\Sigma = \{\texttt{int}, \texttt{+}, \texttt{*}, \texttt{(}, \texttt{)}\} \quad , \quad N = \{E\} \quad , \quad S = E$$

$$P = \{ \; E \rightarrow E\texttt{+}E, \;\; E \rightarrow E\texttt{*}E, \;\; E \rightarrow \texttt{(}E\texttt{)}, \;\; E \rightarrow \texttt{int} \; \}$$

The Language of a CFG

- Given a CFG G, the language $\mathcal{L}(G)$ is defined as the set of all strings that can be *derived* from S
- Given a string α from $(\Sigma \cup N)^*$, derive a new string β:
  - ➤ Choose one of the nonterminals in α, say, A
  - ➤ Choose one of the productions with A on the left hand side
  - ➤ Replace A in α with the right hand side (rhs) of the production to get the derived string β
- If α contains only symbols in Σ, then $\alpha \in \mathcal{L}(G)$
- Example:

  $\alpha =$ `int*`(E); choose $E \rightarrow E$ `+` E; $\beta =$ `int*(`E`+`E`)`

Derivations: Formally

- A string α derives a string β, ($\alpha \underset{G}{\Rightarrow} \beta$) $\alpha, \beta \in (\Sigma \cup N)^*$, if: there are $\gamma, \delta, \eta \in (\Sigma \cup N)^*, A \in N$ such that
  $\alpha = \gamma\, A\, \delta \quad , \quad \beta = \gamma\, \eta\, \delta$ and $A \longrightarrow \eta \in P$
- We write $\alpha \underset{G}{\Rightarrow} \beta$ for a one-step derivation
- $\alpha \overset{*}{\underset{G}{\Rightarrow}} \beta$ is a many-step derivation: $\alpha \underset{G}{\Rightarrow} \alpha_0 \underset{G}{\Rightarrow} \alpha_1 \ldots \underset{G}{\Rightarrow} \beta$
- Language $\mathcal{L}(G)$ generated by G: $\mathcal{L}(G) = \{s \in \Sigma^* | S \overset{*}{\underset{G}{\Rightarrow}} s\}$
- The task of a parser: find one (or all) derivation(s) of a string in Σ^*, given a CFG G

Derivations: Example

- $\Sigma = \{$*john, girl, car, saw, walks, in, the, a*$\}$
- $N = \{$S, NP, VP, PP, D, N, V, P$\}$
- $P = \left\{ \begin{array}{ll} \text{S} \rightarrow \text{NP VP} | \text{N VP} | \text{N V} | \text{NP V} & \text{N} \rightarrow \textit{john, girl, car} \\ \text{VP} \rightarrow \text{V NP} | \text{V N} | \text{VP PP} & \text{V} \rightarrow \textit{saw, walks} \\ \text{NP} \rightarrow \text{D N} | \text{NP PP} | \text{N PP} & \text{P} \rightarrow \textit{in} \\ \text{PP} \rightarrow \text{P NP} | \text{P N} & \text{D} \rightarrow \textit{the, a} \end{array} \right\}$

S $\underset{G}{\Rightarrow}$

john saw the girl in a car

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$\text{S} \underset{G}{\Rightarrow} \text{N VP} \underset{G}{\Rightarrow}$

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$$\text{S} \underset{G}{\Rightarrow} \text{N VP} \underset{G}{\Rightarrow} \text{john VP} \underset{G}{\Rightarrow} \text{john V NP} \underset{G}{\Rightarrow}$$

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S $\underset{G}{\Rightarrow}$ N VP $\underset{G}{\Rightarrow}$ john VP $\underset{G}{\Rightarrow}$ john V NP $\underset{G}{\Rightarrow}$ john saw NP $\underset{G}{\Rightarrow}$
john saw NP PP $\underset{G}{\Rightarrow}$

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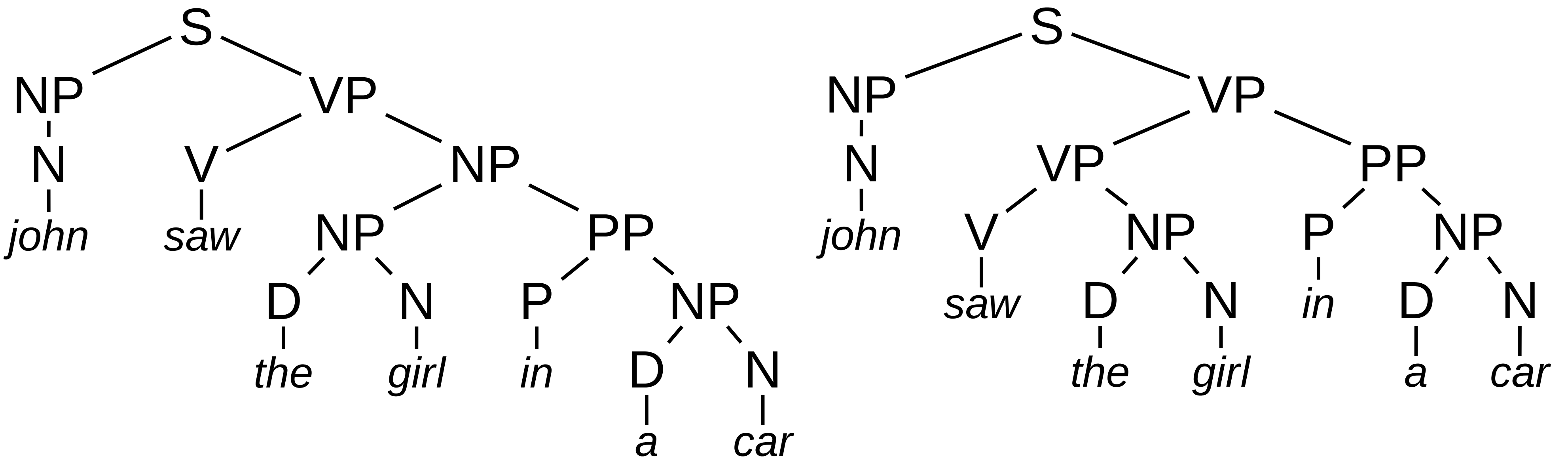
john saw the girl in a car

Derivation (Parse) Trees



- Encodes many possible derivations
- PP node in the example can be attached to two nodes: the grammar is ambigous
- CF Parsers/Recognizers differ in the way the derivation trees are build

Task: given $s \in \Sigma^*$ and G, is $s \in \mathcal{L}(G)$?

Two ways to go:

- start with the start symbol S and try to derive s by systematic application of the productions:
  top down recognition (goal driven)
- start with the string s and try to reduce it to the start symbol:
  bottom up recognition (data driven)

Idea: Recursively compute all expansions of a nonterminal at some input position

$E \rightarrow S$+S, $E \rightarrow S$ * S,
$S \rightarrow$ (E), $S \rightarrow$ int

expand(S, 0)

S

(int + (int * int))

Idea: Recursively compute all expansions of a nonterminal at some input position

expand(S, 0)

$E \longrightarrow S\texttt{+}S, \quad E \longrightarrow S\,\texttt{*}\,S,$

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S

(E)

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$$E \rightarrow S\texttt{+}S, \quad E \rightarrow S\,\texttt{*}\,S,$$
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`expand(`E`, 1)`

S

`(` E `)`

S `*` S

`( int + ( int * int ) )`

Idea: Recursively compute all expansions of a nonterminal at some input position

expand(S, 1)

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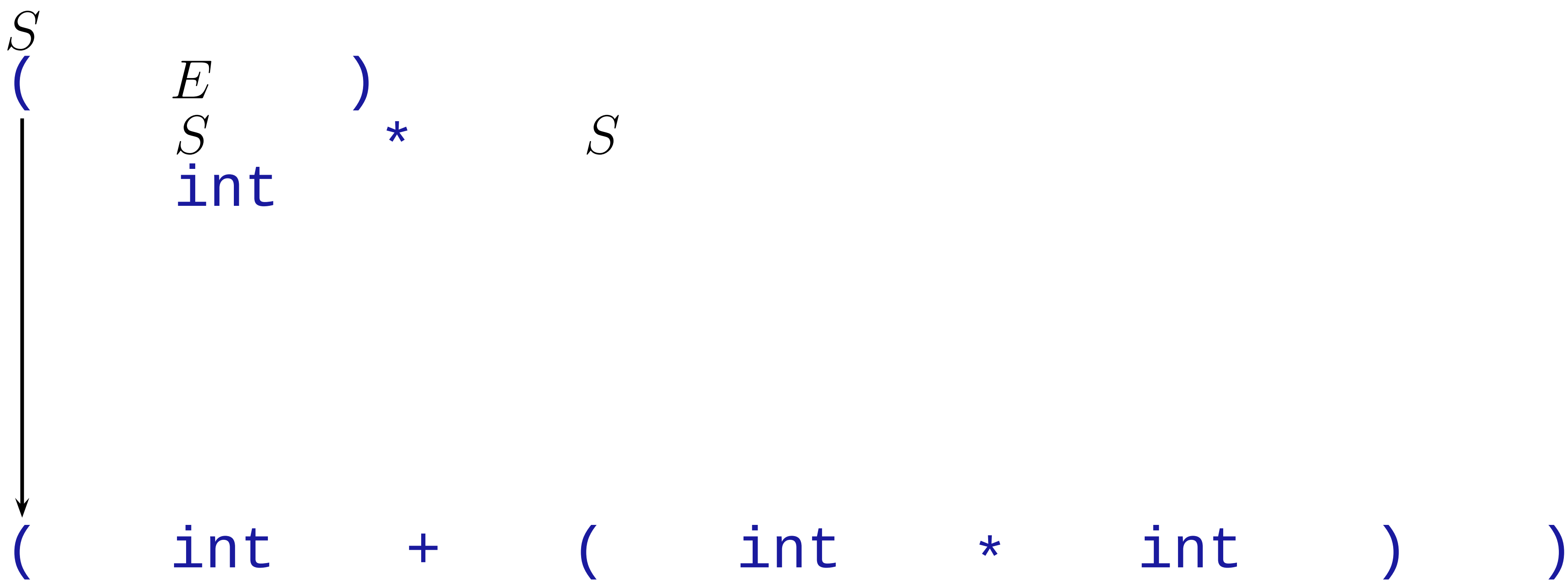

Recursive Descent Parsing

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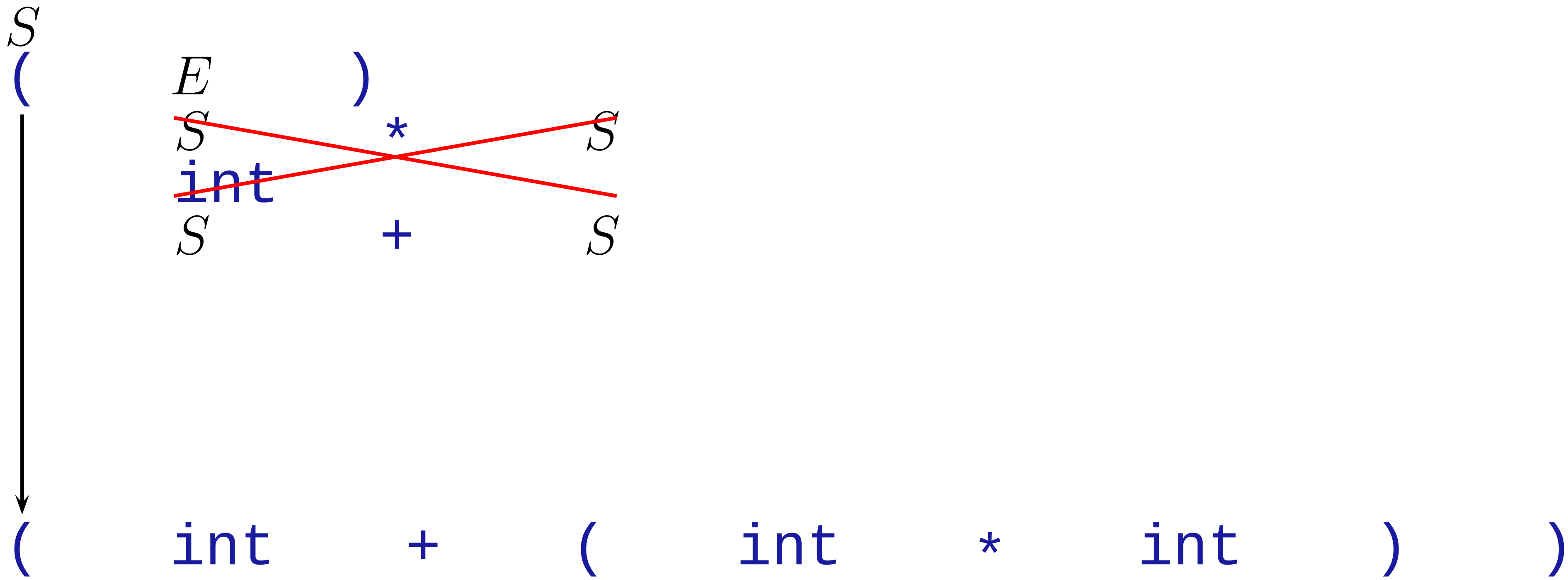

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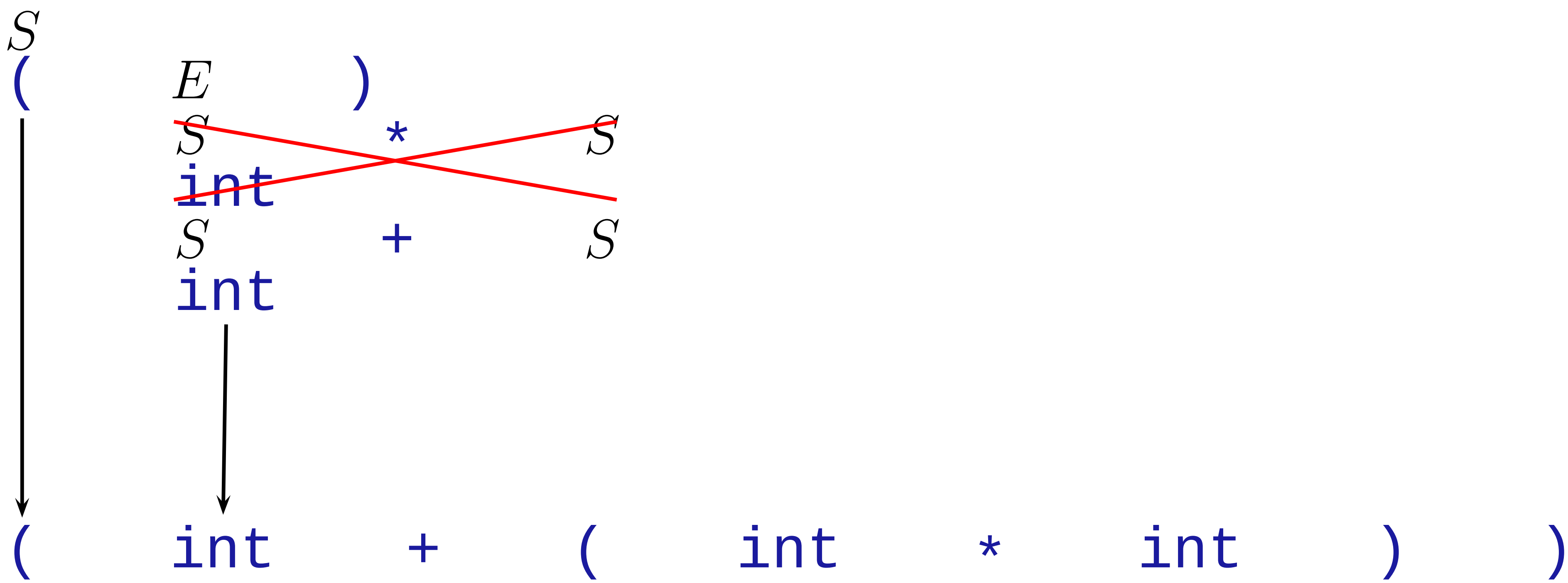

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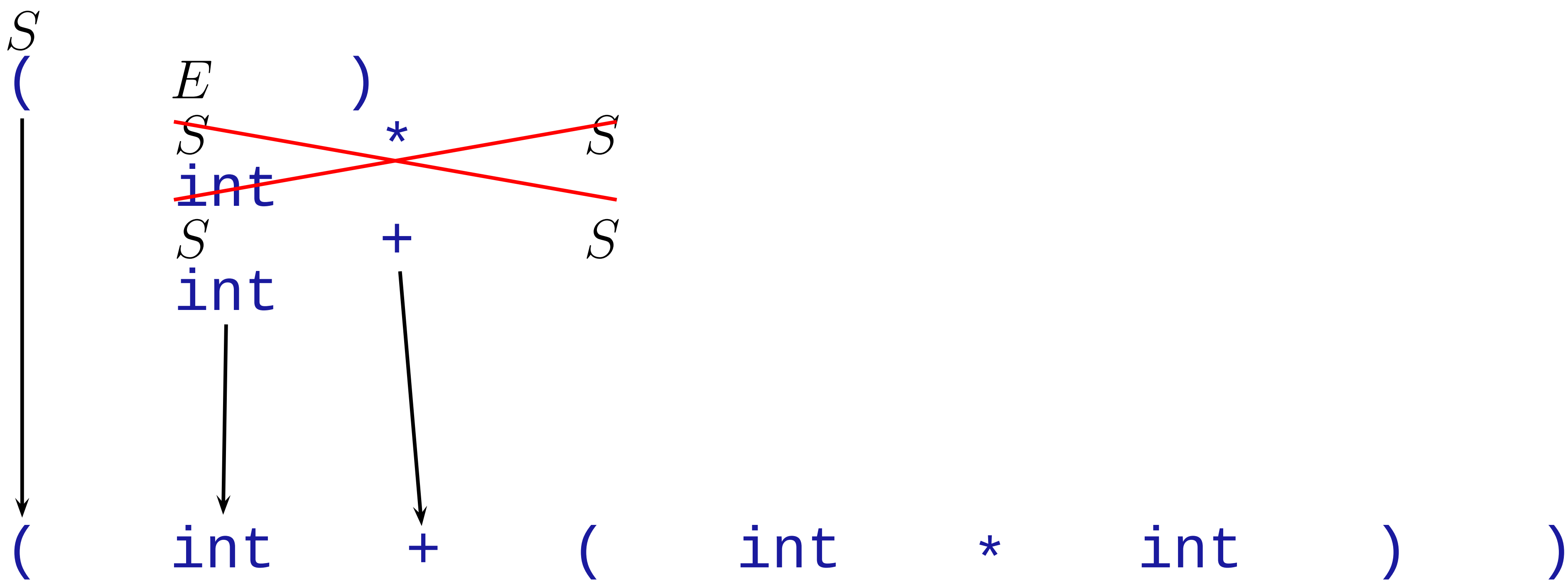

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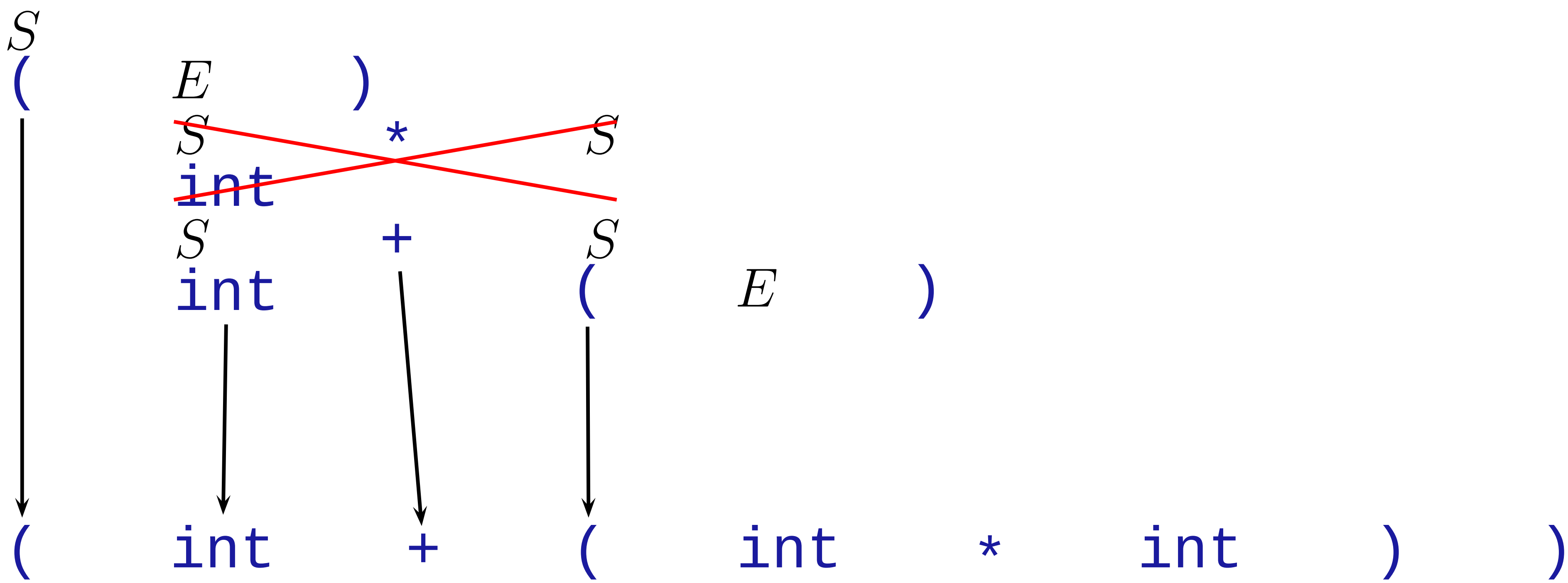
$E \rightarrow S\texttt{+}S,\quad E \rightarrow S\,\texttt{*}\,S,$

$S \rightarrow \texttt{(}\,E\,\texttt{)},\quad S \rightarrow \texttt{int}$

expand(S, 3)

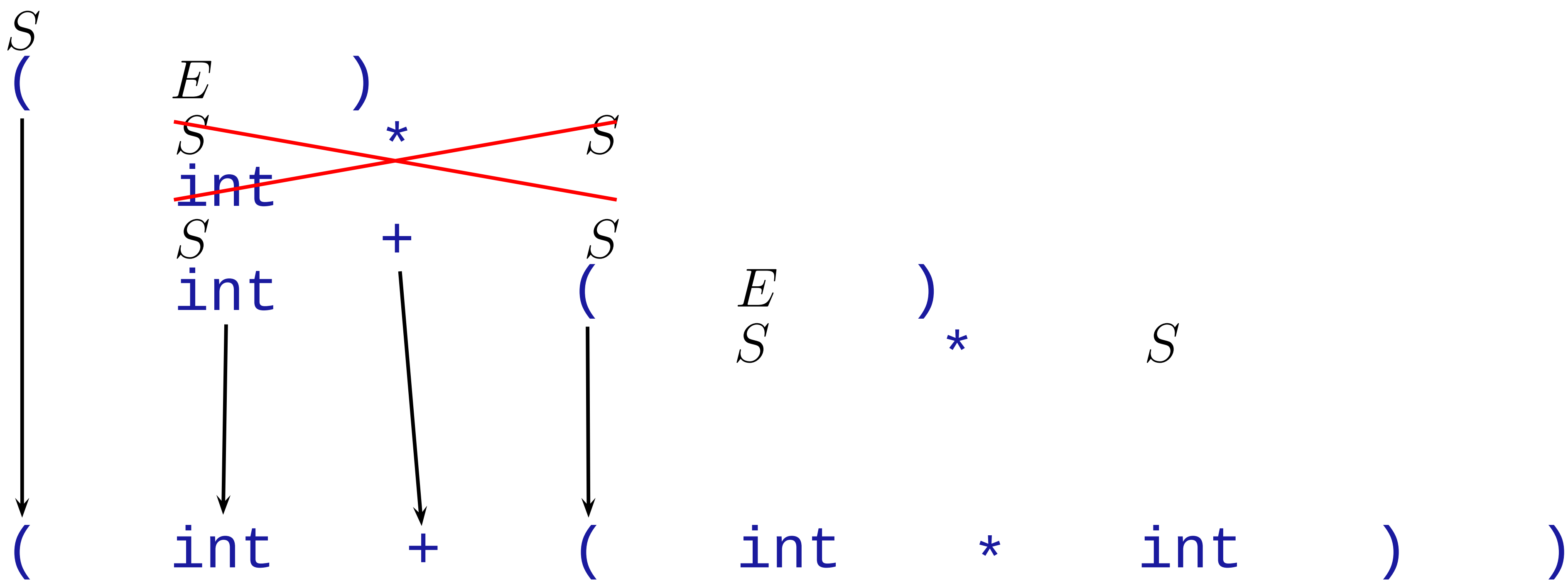


Recursive Descent Parsing

Idea: Recursively compute all expansions of a nonterminal at some input position

`expand(`E`, 4)`

$E \rightarrow S$ `+` S, $E \rightarrow S$ `*` S,
$S \rightarrow$ `(` E `)`, $S \rightarrow$ `int`

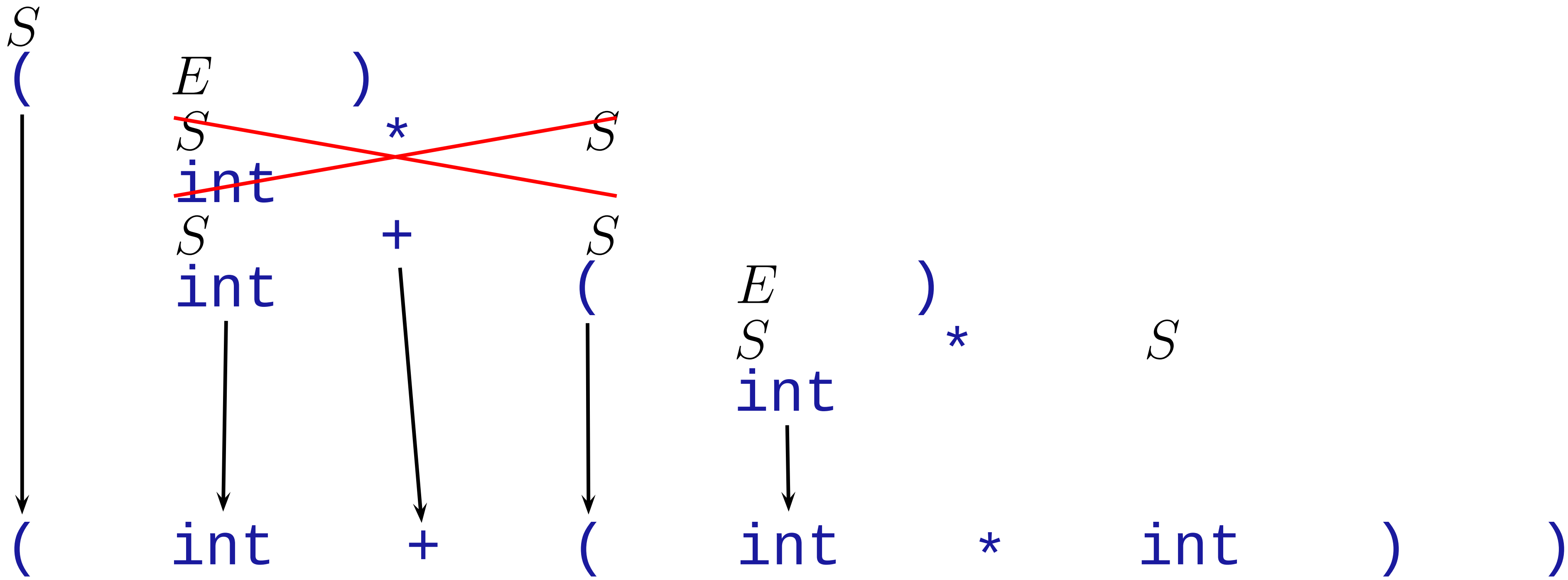


Recursive Descent Parsing

Idea: Recursively compute all expansions of a nonterminal at some input position

expand(S, 4)

$E \to S$ + S, $E \to S$ * S,
$S \to$ (E), $S \to$ int



Recursive Descent Parsing

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$E \longrightarrow S$+S, $E \longrightarrow S$ * S,
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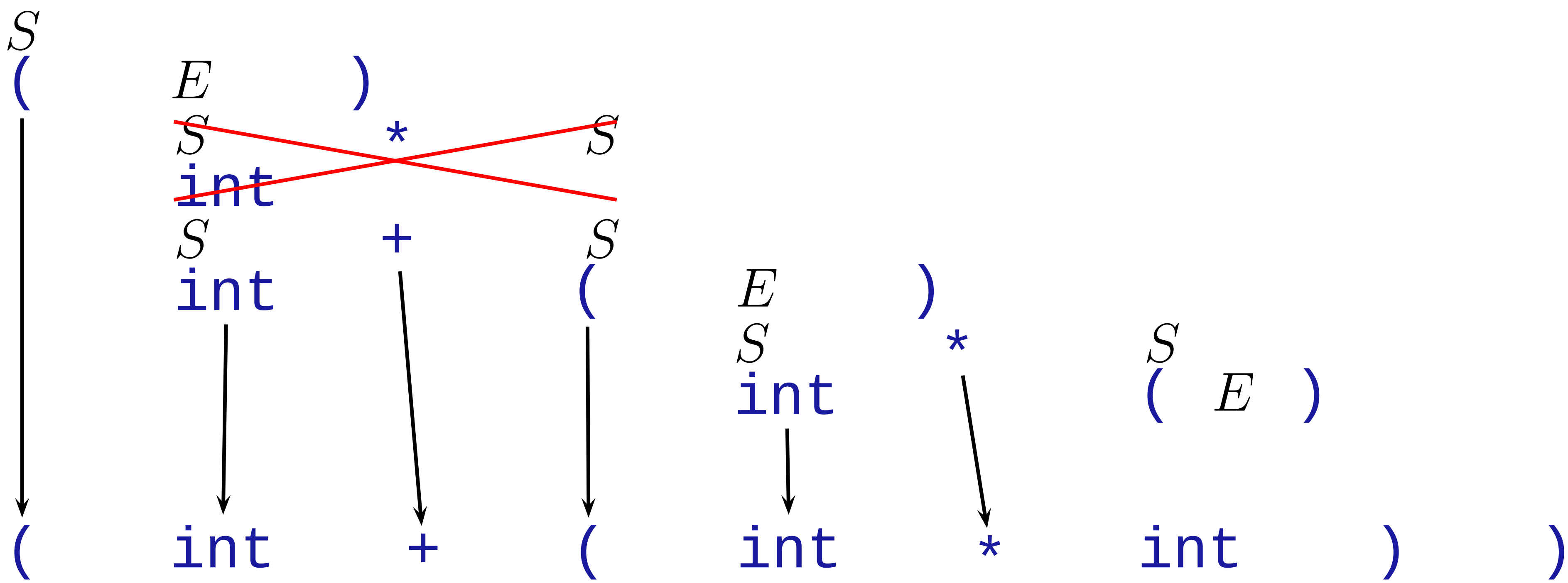
expand(S, 6)



Idea: Recursively compute all expansions of a nonterminal at some input position

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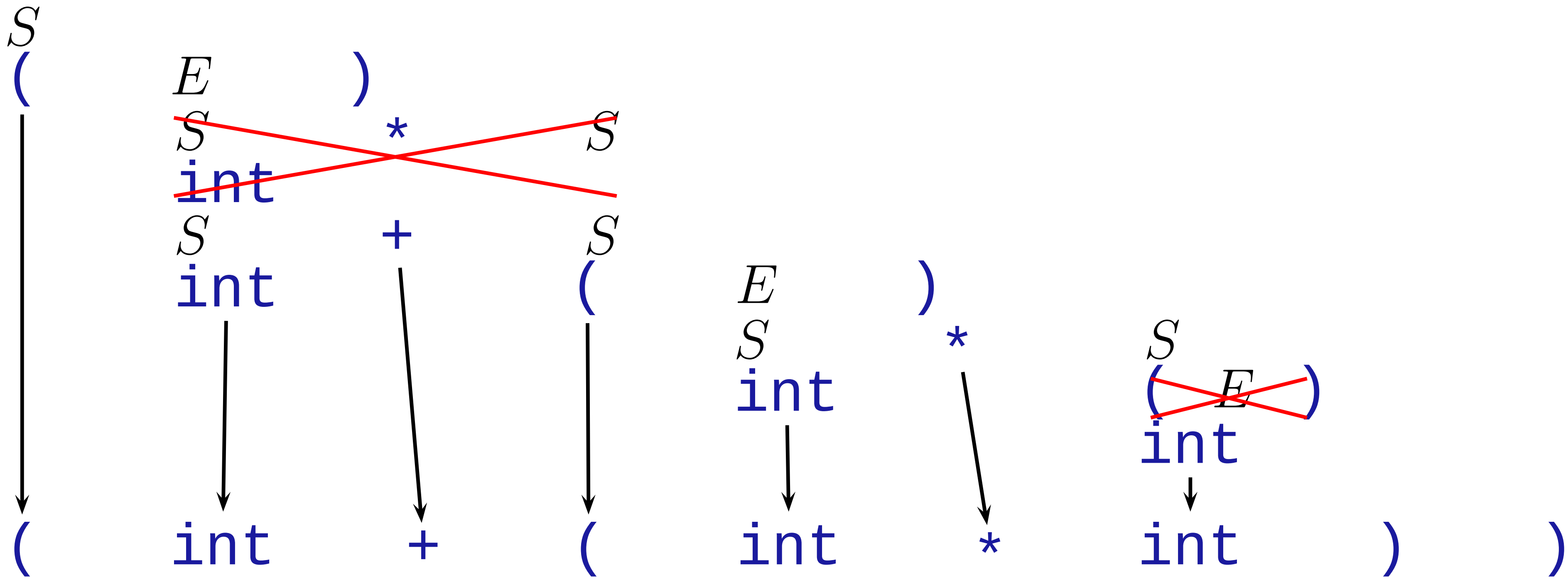
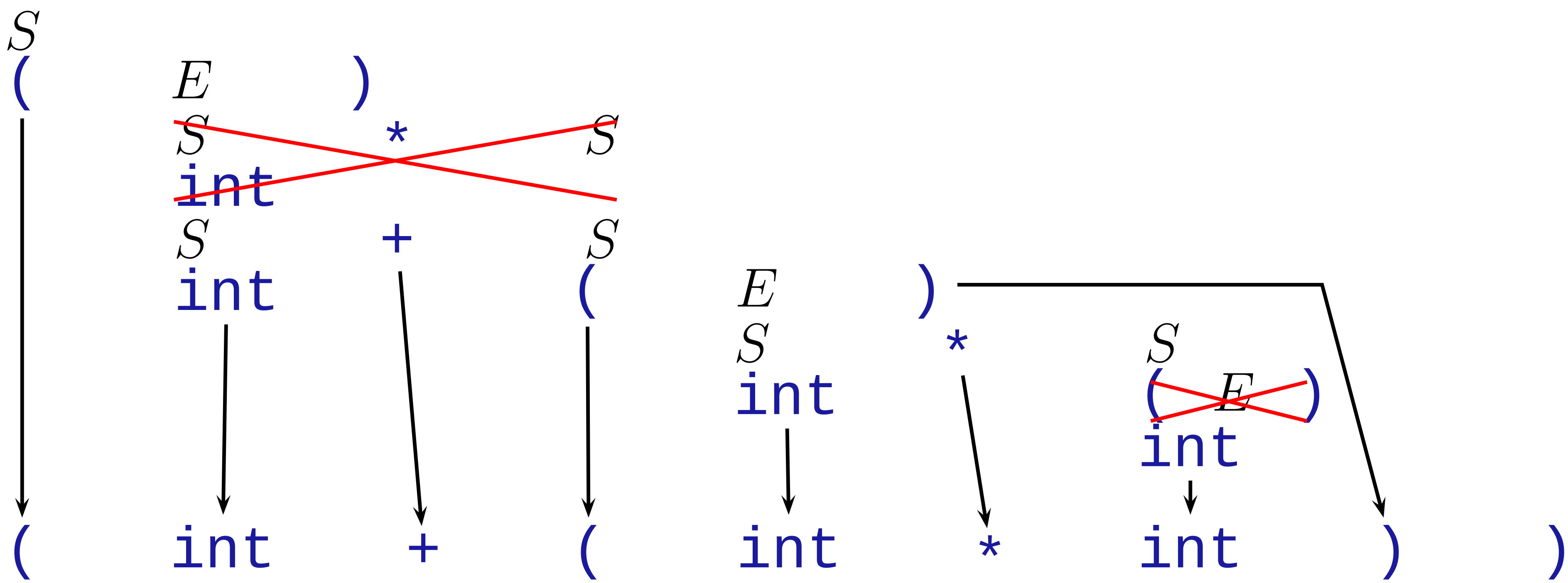


Idea: Recursively compute all expansions of a nonterminal at some input position

expand(S, 3)

$E \longrightarrow S + S, \quad E \longrightarrow S * S,$

$S \longrightarrow (E), \quad S \longrightarrow \texttt{int}$



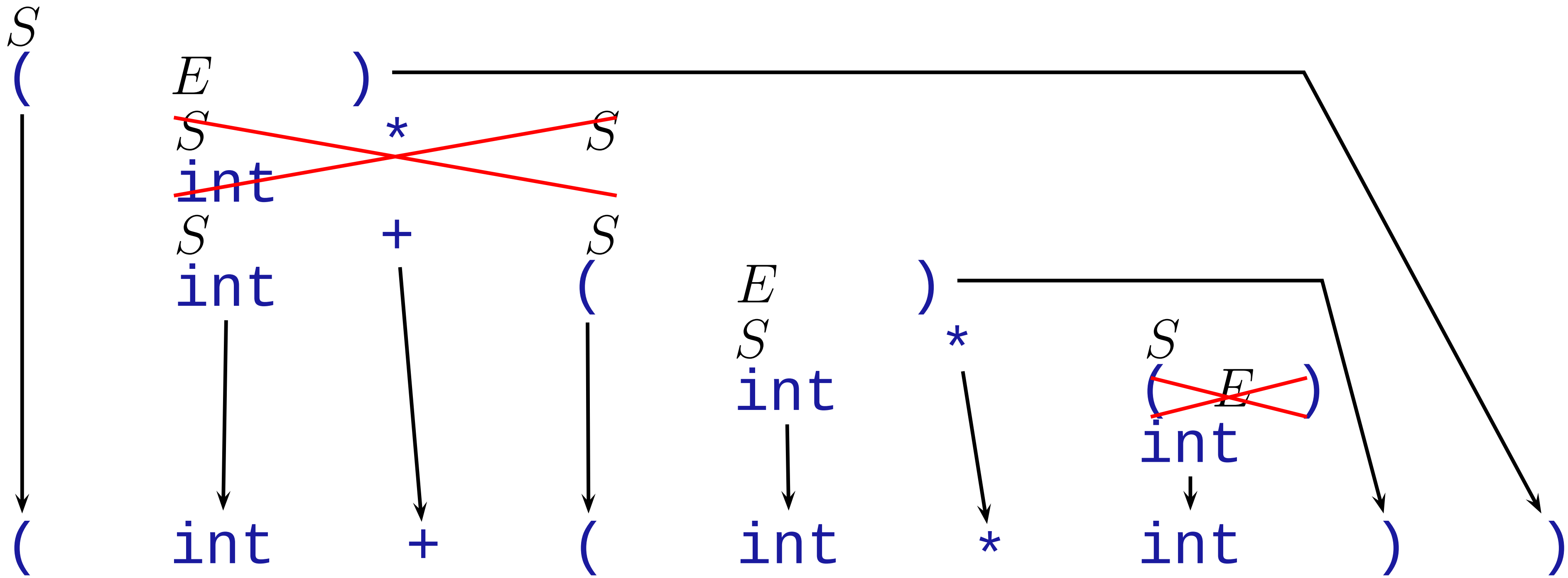
Recursive Descent Parsing

Idea: Recursively compute all expansions of a nonterminal at some input position

`expand(`S`, 0)`

$E \longrightarrow S$ `+` S, $E \longrightarrow S$ `*` S,

$S \longrightarrow$ `(` E `)`, $S \longrightarrow$ `int`



Recursive Descent Parsing II

- expand returns a set of possible end positions
- During expansion of one production
  - Keep track of a set of intermediate positions for the already expanded part
  - Expand the next terminal or nonterminal from every intermediate position
  - If the set of possible intermediate position gets empty, the production fails
- When a production was successfully completed, the intermediate set is added to the set of end positions
- May run into an infinite loop with left-recursive grammars, i.e. grammars where $A \overset{*}{\underset{G}{\Rightarrow}} A\beta$ for some A

Top Down Parsing: Optimizations

- For every nonterminal, compute the set of terminals that can occur in the *first* position
  - ➢ Expand only those nonterminals / productions that are compatible with the current input symbol
- Avoid performing duplicate calls with identical nonterminal/position pair: *memoize* previous calls
  - ➢ use a data structure whose index are tuples consisting of the function arguments
  - ➢ the result of the lookup is the result of a previous call with the same arguments (if available)
  - ➢ The memoized method has to be strictly functional for this to work

Epsilon Reduction

- A CFG may contain productions of the form $A \to \epsilon$
- Construct a CFG G' with the same language as G and at most one epsilon production: $S \to \epsilon$

for all nonterminals A with $A \to \epsilon \in P$:
  mark A as ϵ-deriving and add it to the set Q
while Q is not empty, remove a nonterminal X from Q:
  for all $Y \to \alpha\, X\, \beta \in P$, with α or β not empty, add $Y \to \alpha\, \beta$ to P'
  for all $Y \to X$, if Y is not marked as ϵ-deriving:
    mark Y as ϵ-deriving and add it to Q
if S is ϵ-deriving, add $S \to \epsilon$ to P'
add all non-ϵ productions of P to P'

Chomsky Normal Form

- A context-free grammar is in *Chomsky Normal Form* if:
  (i) it is ϵ-free,
  (ii) all productions have one of two forms:
  - $A \rightarrow a$ with $a \in \Sigma$
  - $A \rightarrow B\,C$ with $B, C \in N$
- Every CFG can be transformed into a CNF grammar with the same language
- Drawback: The original structure of the parse trees must be reconstructed, if necessary
- The original and transformed grammar are said to be *weakly equivalent*

Chomsky Normal Form II

- Convert an arbitrary CFG into CNF:
  - Introduce new nonterminals and productions $A^a \rightarrow a$
  - Replace all occurences of a by A^a
  - Eliminate unary productions $A \rightarrow B$:
    add productions where A is replaced by B in the right hand sides
  - Replace productions with more than two symbols on the right hand side by a sequence of productions:
    $A \rightarrow R_1\, R_2\, \ldots R_n \Rightarrow$
    $$A \rightarrow R_1 A^{(1)}, \quad A^{(1)} \rightarrow R_2 A^{(2)}, \quad \cdots \quad A^{(n-2)} \rightarrow R_{n-1} R_n$$

Chart Parsing

- First algorithm independently developed by Cocke, Younger and Kasami (late 60s)
- Given a string w of length n, use an $n \times n$ table to store subderivations (hence chart or tabular parsing)
- Works for all kinds of grammars: left/right recursive, ambiguous
- Storing subderivations avoids duplicate computation: an instance of *dynamic programming*
- polynomial space and time bounds, although an exponential number of parse trees may be encoded!

- Input: G in Chomsky normal form, input string $w_1, \ldots, w_n$
- Systematically explore all possible sub-derivations bottom-up
- Use an $n \times n$ array $\mathcal{C}$ such that
  - ➤ If nonterminal A is stored in $\mathcal{C}(i,k)$: $A \overset{*}{\underset{G}{\Rightarrow}} w_{i+1}, \ldots, w_k$
  - ➤ Maintain a second table $\mathcal{B}$, such that if $j \in \mathcal{B}(i,k)$: ex. $A \rightarrow B\,C \in P, A \in \mathcal{C}(i,k), B \in \mathcal{C}(i,j)$ and $C \in \mathcal{C}(j,k)$
  - ➤ $\mathcal{B}$ enables us to extract the parse trees
- Implement $\mathcal{C}$ and $\mathcal{B}$ as three-dimensional boolean arrays of size $n \times n \times |N|$ and $n \times n \times n$, respectively

For $i = 1$ to n

    For each $R_j \longrightarrow a_i$, set $\mathcal{C}(i-1, i, j) = \texttt{true}$

For $l = 2$ to n *– Length of new constituent*

    For $i = 0$ to $n - l$ *– Start of new constituent*

        For $m = 1$ to $l - 1$ *– Length of first subconstituent*

            For each production $R_a \longrightarrow R_b R_c$

                If $\mathcal{C}(i, i+m, b)$ and $\mathcal{C}(i+m, i+l, c)$ then

                    set $\mathcal{C}(i, i+l, a) = \texttt{true}$

                    set $\mathcal{B}(i, i+l, i+m) = \texttt{true}$

If $\mathcal{C}(1, n, S)$ is `true`, $w \in \mathcal{L}(G)$

CYK Chart Example

$\mathcal{C}$

$\mathcal{B}$

⓪ *john* ① *saw* ② *the* ③ *girl* ④ *in* ⑤ *a* ⑥ *car* ⑦

CYK Chart Example

$\mathcal{C}$

N						
	V					
		D				
			N			
				P		
					D	
						N

$\mathcal{B}$

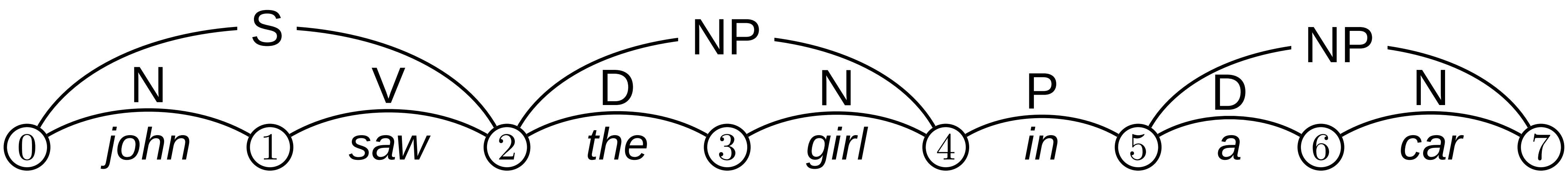
CYK Chart Example

$\mathcal{C}$

N	S					
	V					
		D	NP			
			N			
				P		
					D	NP
						N

$\mathcal{B}$

	1					
			3			
						6



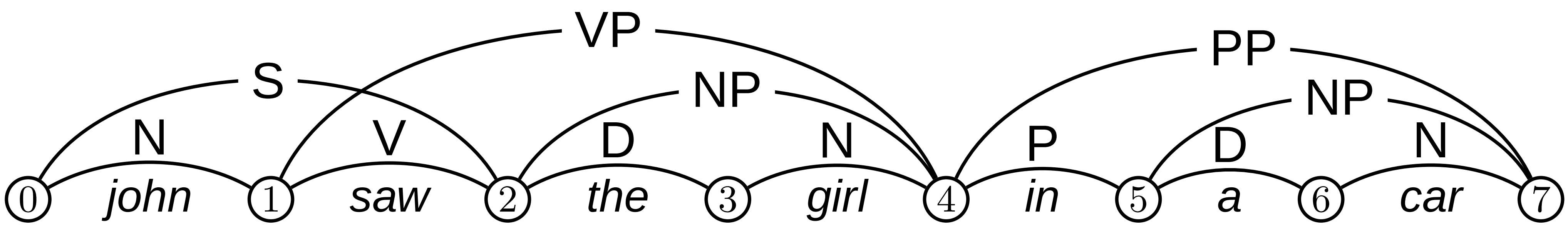
CYK Chart Example

$\mathcal{C}$

N	S					
	V		VP			
		D	NP			
			N			
				P		PP
					D	NP
						N

$\mathcal{B}$

	1					
			2			
			3			
						5
						6



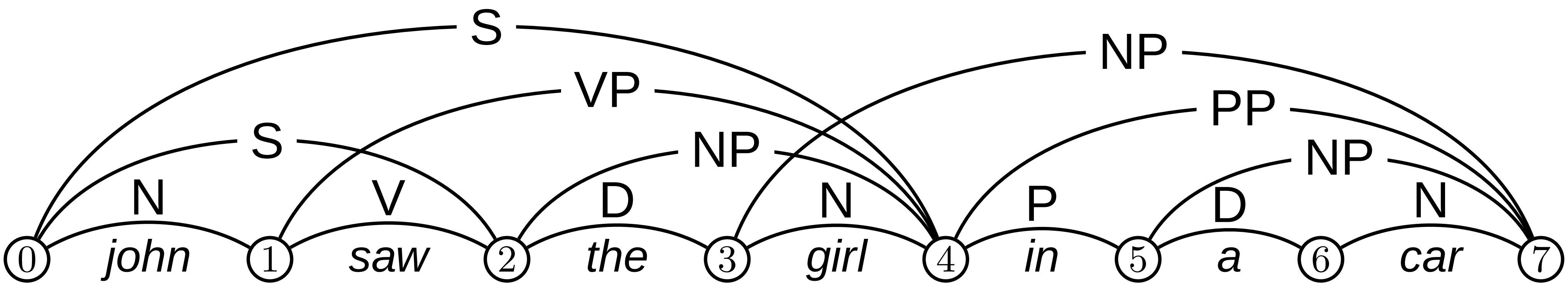
CYK Chart Example

$\mathcal{C}$

N	S		S			
	V		VP			
		D	NP			
			N			NP
				P		PP
					D	NP
						N

$\mathcal{B}$

	1		1			
			2			
			3			
						4
						5
						6



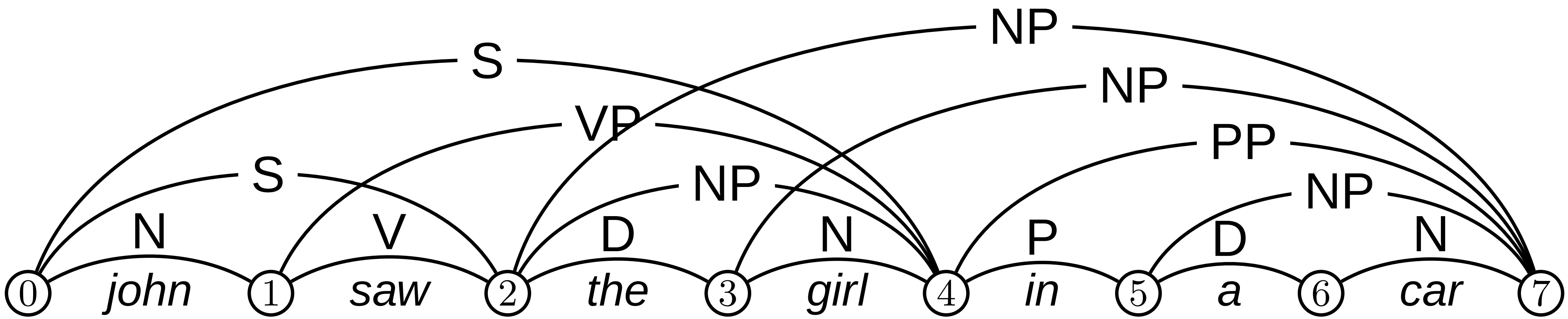
CYK Chart Example

$\mathcal{C}$

N	S		S			
	V		VP			
		D	NP			NP
			N			NP
				P		PP
					D	NP
						N

$\mathcal{B}$

	1		1			
			2			
			3			4
						4
						5
						6



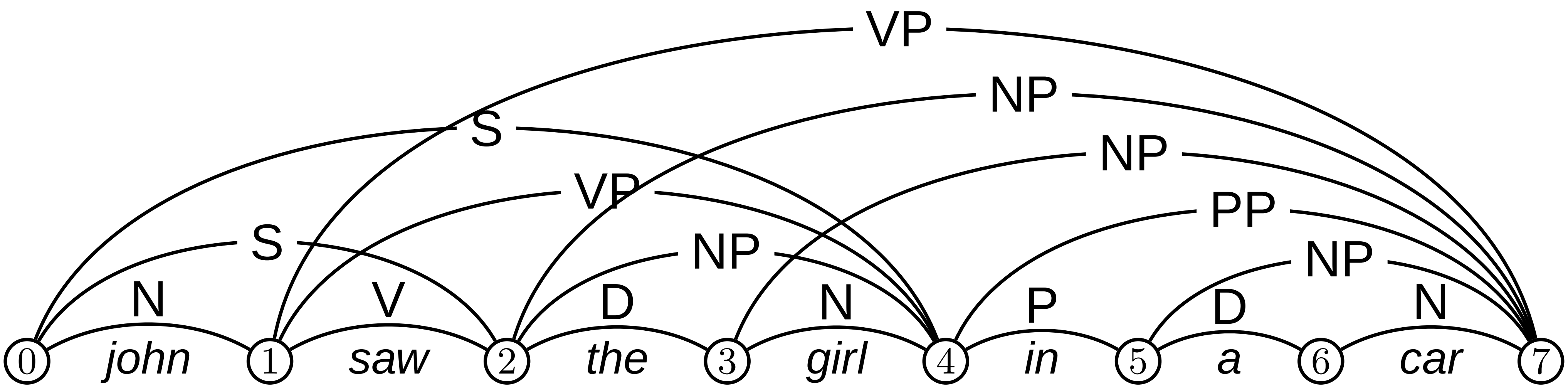
CYK Chart Example

$\mathcal{C}$

N	S		S			
	V		VP			VP
		D	NP			NP
			N			NP
				P		PP
					D	NP
						N

$\mathcal{B}$

	1		1			
			2			2,4
			3			4
						4
						5
						6



CYK Chart Example

$\mathcal{C}$

N	S		S			S
	V		VP			VP
		D	NP			NP
			N			NP
				P		PP
					D	NP
						N

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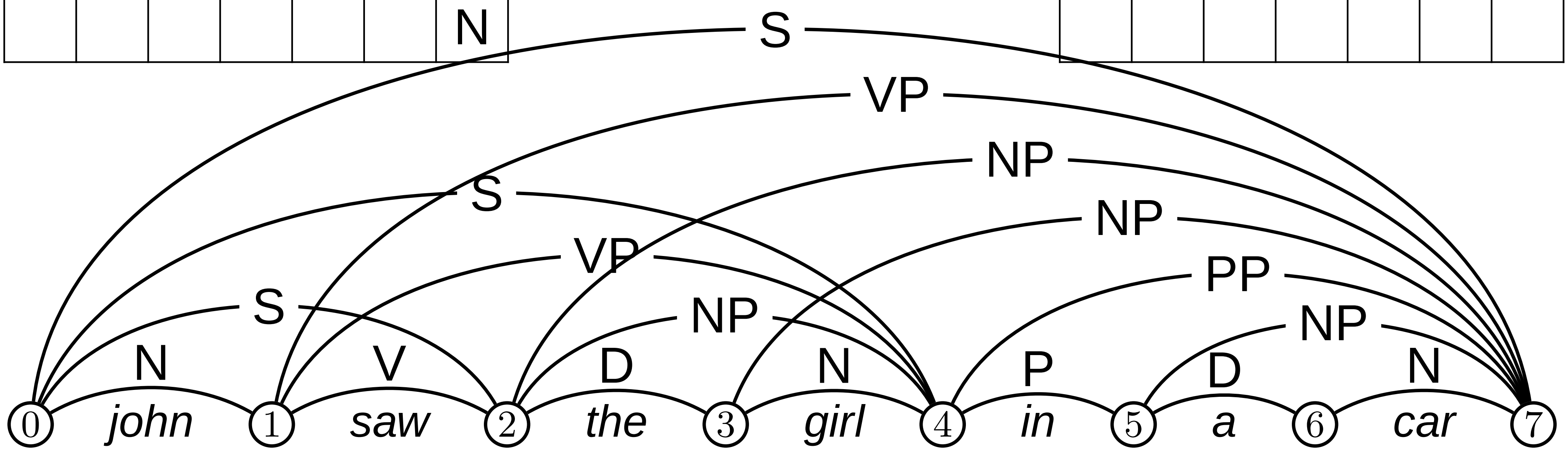
	1		1			1
			2			2,4
			3			4
						4
						5
						6



Example Chart

S →NP VP|N VP|N V|NP V

VP→V NP|V N|VP PP

NP→D N|NP PP|N PP

PP→P NP|P N

N →*john, girl, car*

V →*saw, walks*

P →*in*

D →*the, a*

	1	2	3	4	5	6	7
0	N	S		S			S
1		V		VP			VP(2)
2			D	NP			NP
3				N			NP
4					P		PP
5						D	NP
6							N

$_0$ *john* $_1$ *saw* $_2$ *the* $_3$ *girl* $_4$ *in* $_5$ *a* $_6$ *car* $_7$

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0	N	S		S			S
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	1	2	3	4	5	6	7
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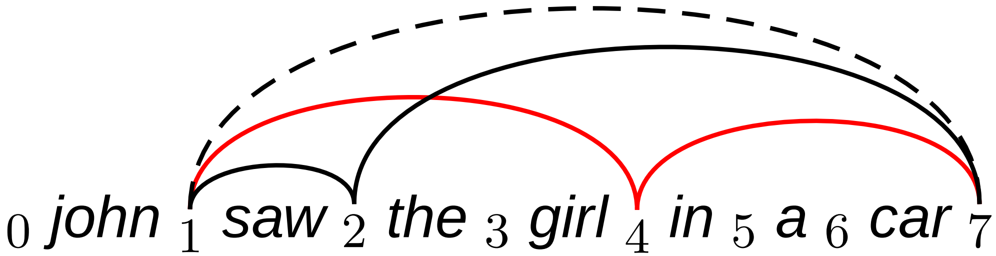
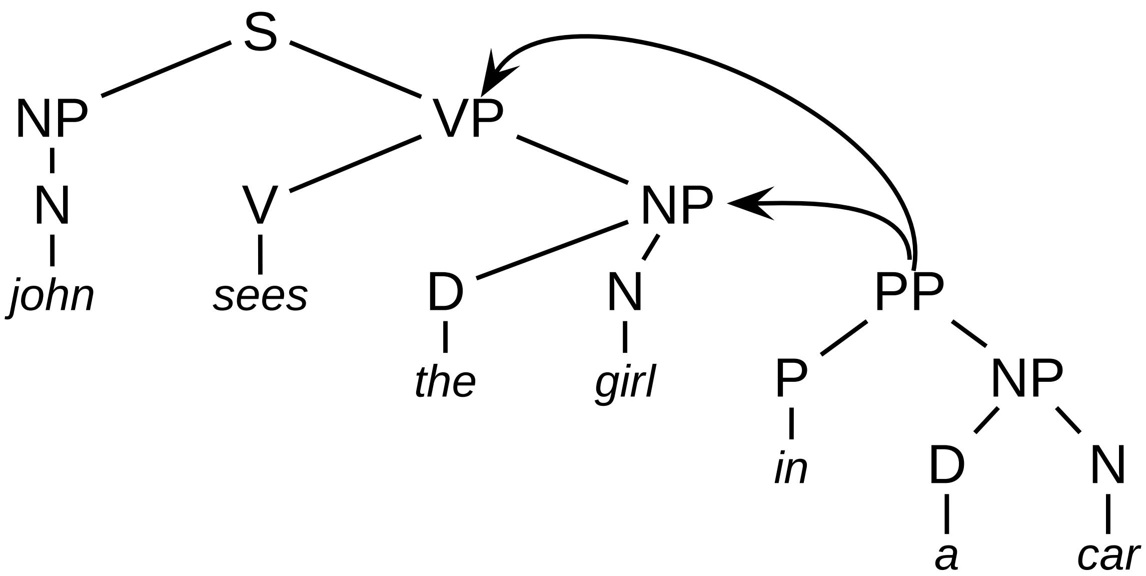


Encoding Ambiguities

$\Sigma = \{$*john, girl, car, sees, in, the, a*$\}$ $N = \{$S, NP, VP, PP, D, N, V, P$\}$

$$P = \left\{ \begin{array}{ll} \text{S} \rightarrow \text{NP VP,} & \text{N} \rightarrow \textit{john, girl, car} \\ \text{VP} \rightarrow \text{V} | \text{V NP} | \text{V NP PP} & \text{V} \rightarrow \textit{sees} \\ \text{NP} \rightarrow \text{N} | \text{D N} | \text{N PP} | \text{D N PP} & \text{P} \rightarrow \textit{in} \\ \text{PP} \rightarrow \text{P NP} & \text{D} \rightarrow \textit{the, a} \end{array} \right\}$$



Describing Chart Parsing Algorithms

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- They are described using chart items, which consist of
  - ➤ a symbol, derived from the grammar
  - ➤ a start and end position from $0, \ldots, n$
- The symbol of *complete* items is one of $\Sigma \cup N$
- *Incomplete* chart items encode partially filled rules
  - ➤ the symbol is a pair (r, i) of rule and *dot position* if $P \ni r : A \rightarrow \alpha\beta$ with $|\alpha| = i$
  - ➤ write alternatively: $A \rightarrow \alpha \bullet \beta$

Bottom-Up Chart Parsing

- How, when and which chart items are created or combined characterizes a parsing algorithm or parsing strategy
- First: A modified variant of Cocke-Younger-Kasami (CYK) algorithm
- Prerequisites: CFG G, input string $w = a_1, \ldots, a_n$
- Data Structures:
  - ➤ A $n+1 \times n+1$ chart $\mathcal{C}$, where each cell contains a set of (complete or incomplete) chart items
  - ➤ A set of chart items $\mathcal{A}$ (those must still be treated in some way)
- Initialization: add all $(a_i, i-1, i)$ to $\mathcal{A}$ and $\mathcal{C}_{i-1,i}$

Bottom-Up Parsing Algorithm

while $\mathcal{A}$ not empty
  take an (X, i, j) from $\mathcal{A}$ and remove it
  if $X \in \Sigma \cup N$
    for $P \ni r \equiv A \rightarrow X\alpha$ do
      check_and_add$(A \rightarrow X \bullet \alpha, i, j)$
    for $k \in 0, \ldots, i-1$ do
      for all $(A \rightarrow \beta \bullet X\alpha, k, i) \in \mathcal{C}$ do
        check_and_add$(A \rightarrow \beta X \bullet \alpha, k, j)$
  else   – incomplete item: $X \equiv A \rightarrow \beta \bullet Y\alpha$
    for $k \in j+1, \ldots, n$ do
      if $(Y, j, k) \in \mathcal{C}$ *check_and_add*$(A \rightarrow \beta Y \bullet \alpha, i, k)$

check_and_add$(X \equiv A \rightarrow \alpha \bullet \beta, i, j) \equiv$
  if $\beta = \epsilon$ then if $(A, i, j) \notin \mathcal{C}$ add (A, i, j) to $\mathcal{A}$ and $\mathcal{C}$ endif
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Bottom-Up: The Details

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  - Operations: add , get and remove some element
  - (priority) queue, stack
  - $\mathcal{A}$ is called *agenda* and can be used to implement search strategies
- Keep terminal items separate from the chart for space and time efficiency

Runtime of CYK Algorithm

check_and_add$(X \equiv A \rightarrow \alpha\bullet\beta, i, j) \equiv$

if $\beta = \epsilon$ then if $(A, i, j) \notin \mathcal{C}$ add (A, i, j) to $\mathcal{A}$ and $\mathcal{C}$ endif
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for $k \in j+1, \ldots, n$ do — max. n times

if $(Y, j, k) \in \mathcal{C}$ *check_and_add*$(A \rightarrow \beta Y \bullet \alpha, i, k)$

CYK – Summary

- Polynomial complexity: $\mathcal{O}(|G|^2 n^3)$
- Explores all possible sub-derivations
- Advantageous for *robust parsing*:
  Extract the biggest/best chunks for ungrammatical input
- That a derivation must start at S is not used at all
  - ➤ Average time is near or equal to the worst case
  - ➤ May lead to poor performance in practice
- Two main steps:
  - ➤ if $(X, i, j) \in \mathcal{C}, X \in \Sigma \cup N$ and $A \rightarrow X\alpha \in P$:
    add $(A \rightarrow X \bullet \alpha, i, j)$ to $\mathcal{C}$
  - ➤ if $(A \rightarrow \beta \bullet Y \alpha, i, j) \in \mathcal{C}$ and $(Y, j, k) \in \mathcal{C}$:
    add $(A \rightarrow \beta Y \bullet \alpha, i, k)$ to $\mathcal{C}$

Predictive Bottom-Up: Earley Parsing

- Described by J. Earley (1970): Predict Top-Down and Complete Bottom-Up
- Initialize by adding the terminal items *and* $(S \rightarrow \alpha, 0, 0)$ for all $S \rightarrow \bullet\alpha \in P$
- Three main operations:

Prediction if $(A \rightarrow \beta \bullet Y\alpha, i, j) \in \mathcal{C}$, add $(Y \rightarrow \bullet\gamma, j, j)$ to $\mathcal{C}$ for every $Y \rightarrow \gamma \in P$

Scanning if $(A \rightarrow \beta \bullet a_{j+1}\alpha, i, j) \in \mathcal{C}$, add $(A \rightarrow \beta a_{j+1} \bullet \alpha, i, j+1)$ to $\mathcal{C}$

Completion if $(Y, i, j), Y \in N$ and $(A \rightarrow \beta \bullet Y\alpha, j, k) \in \mathcal{C}$, add $(A \rightarrow \beta Y \bullet \alpha, i, k)$

Earley Parsing Example

Earley Parsing Example

○ — *john* — ○ — *saw* — ○ — *the* — ○ — *girl* — ○ — *in* — ○ — *a* — ○ — *car* — ○

S → • NP VP

Earley Parsing Example

○ *john* ○ *saw* ○ *the* ○ *girl* ○ *in* ○ *a* ○ *car* ○

S→ • NP VP
NP→ • D N PP
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NP → • D N
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N → • *girl*
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D → • *a*

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N → • *car*

D → • *the*

D → • *a*

Earley Parsing Example

S → • NP VP

NP → • D N PP

NP → • N

NP → • D N

NP → • N PP

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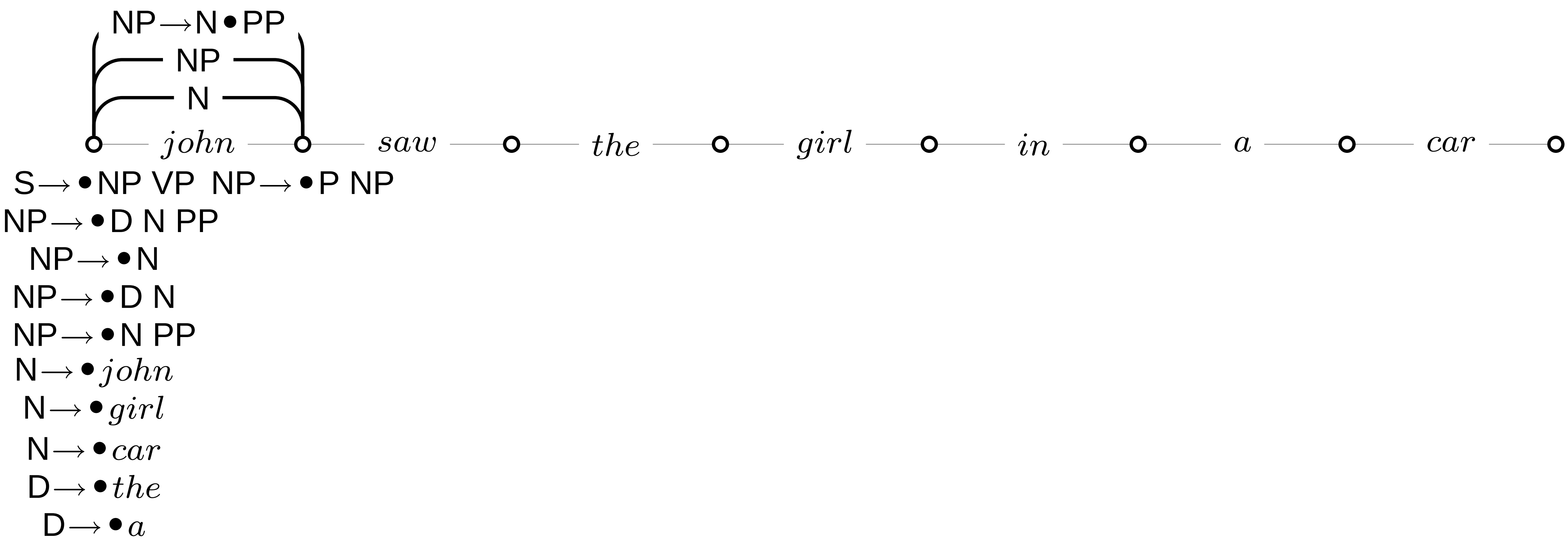

NP→N•PP
NP
N
john
saw
the
girl
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a
car
S→•NP VP
NP→•P NP
NP→•D N PP
NP→•N
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N→•girl
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D→•the
D→•a

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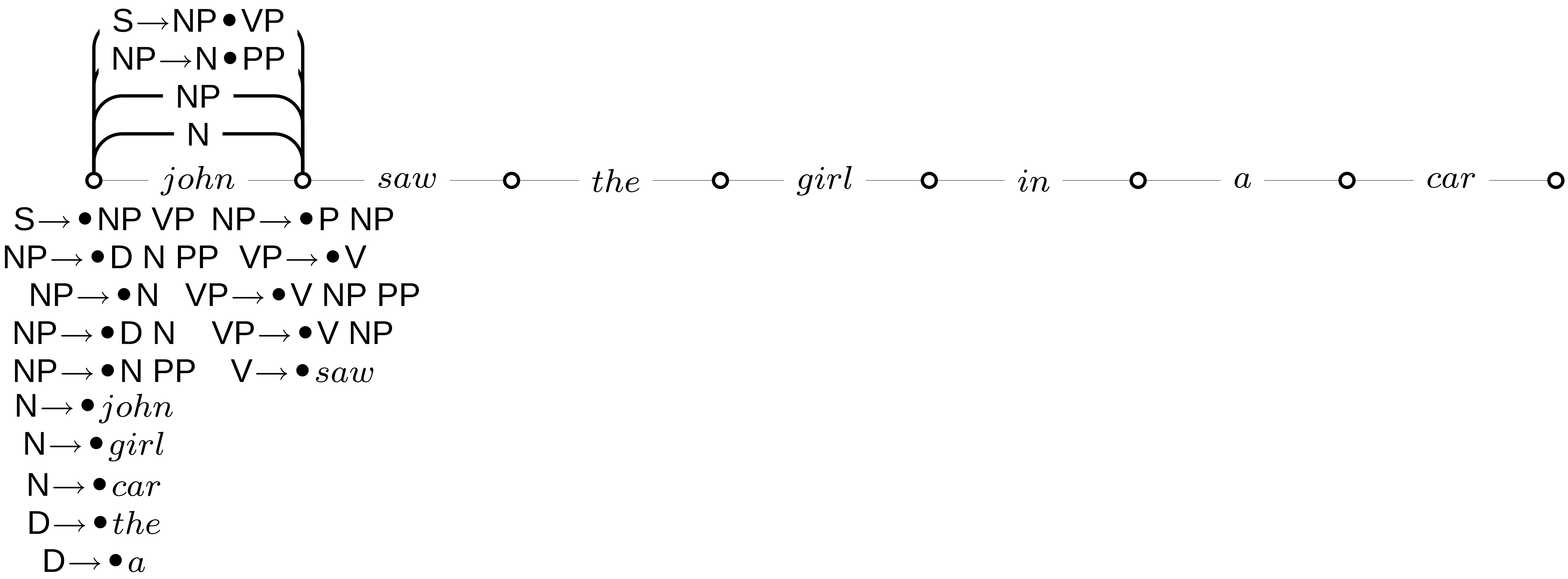

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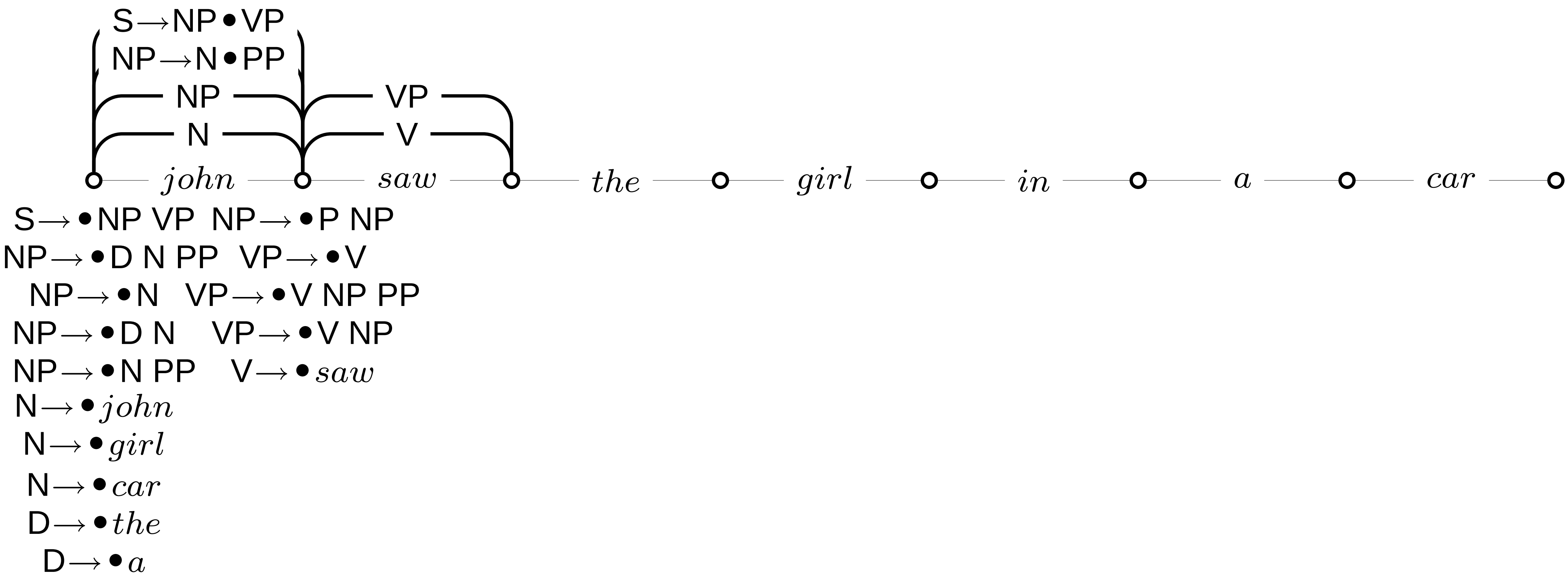

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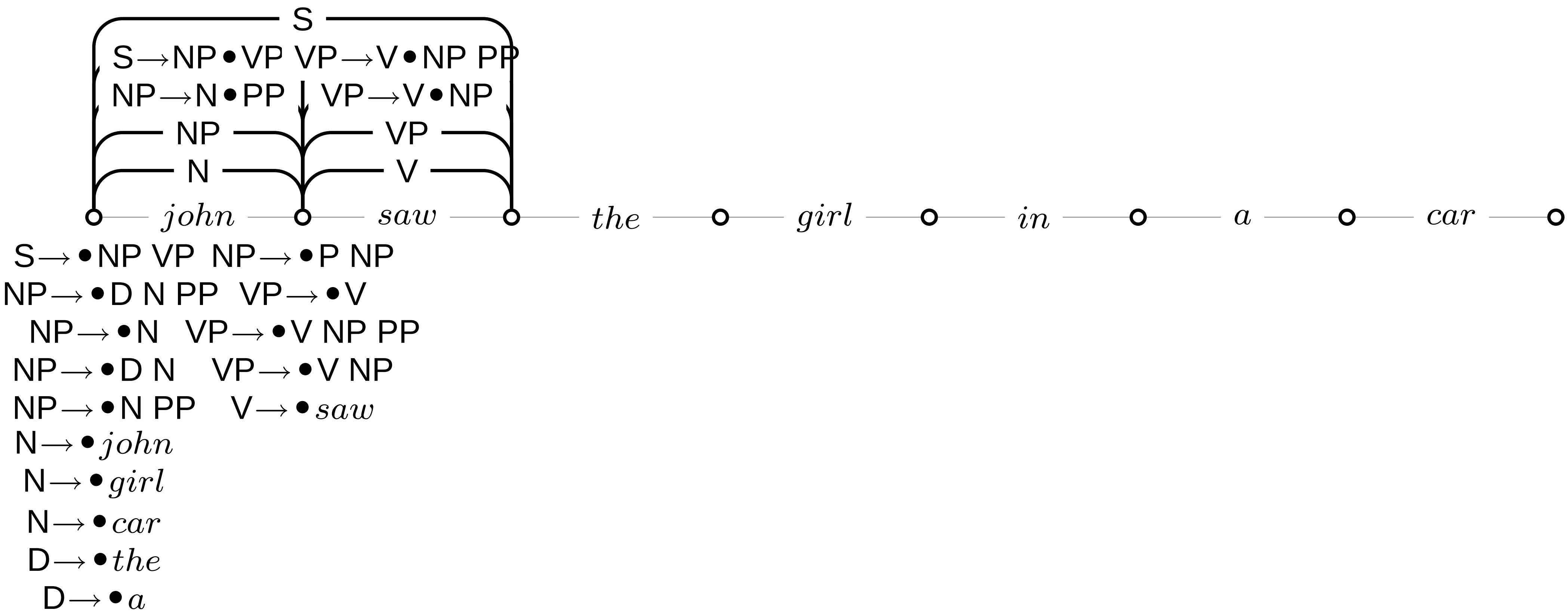

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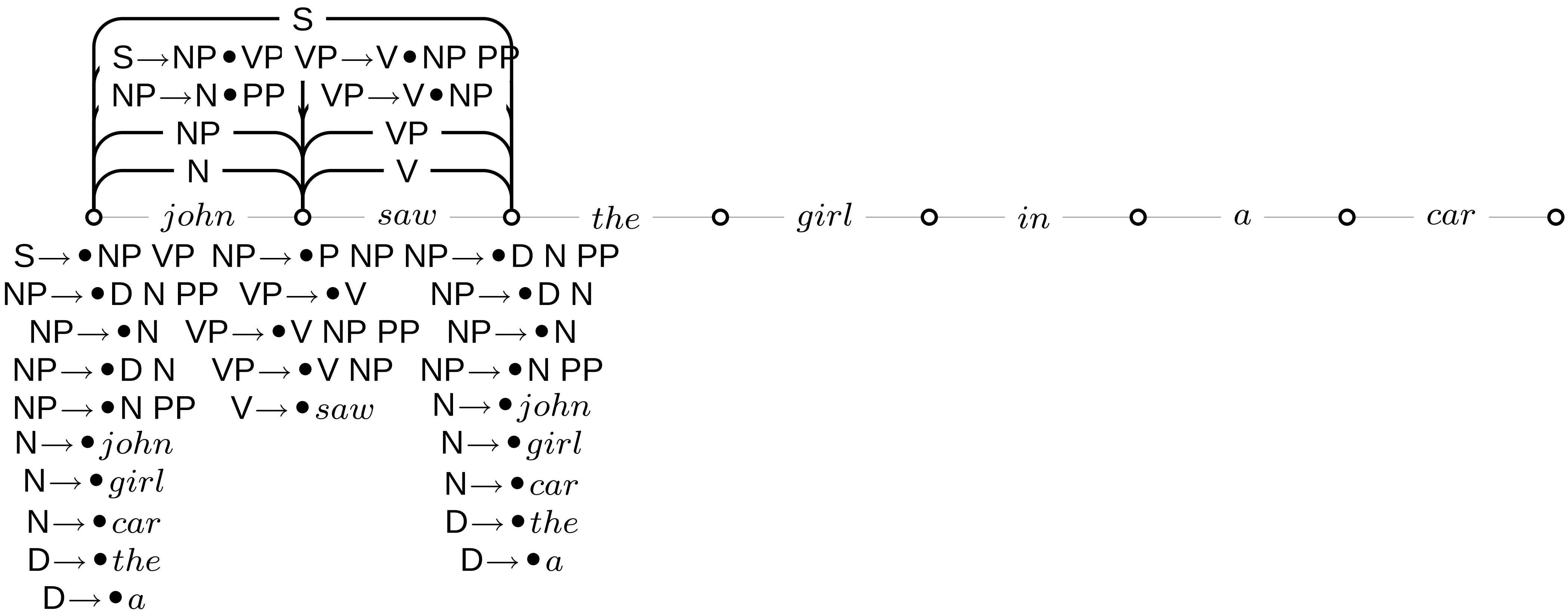

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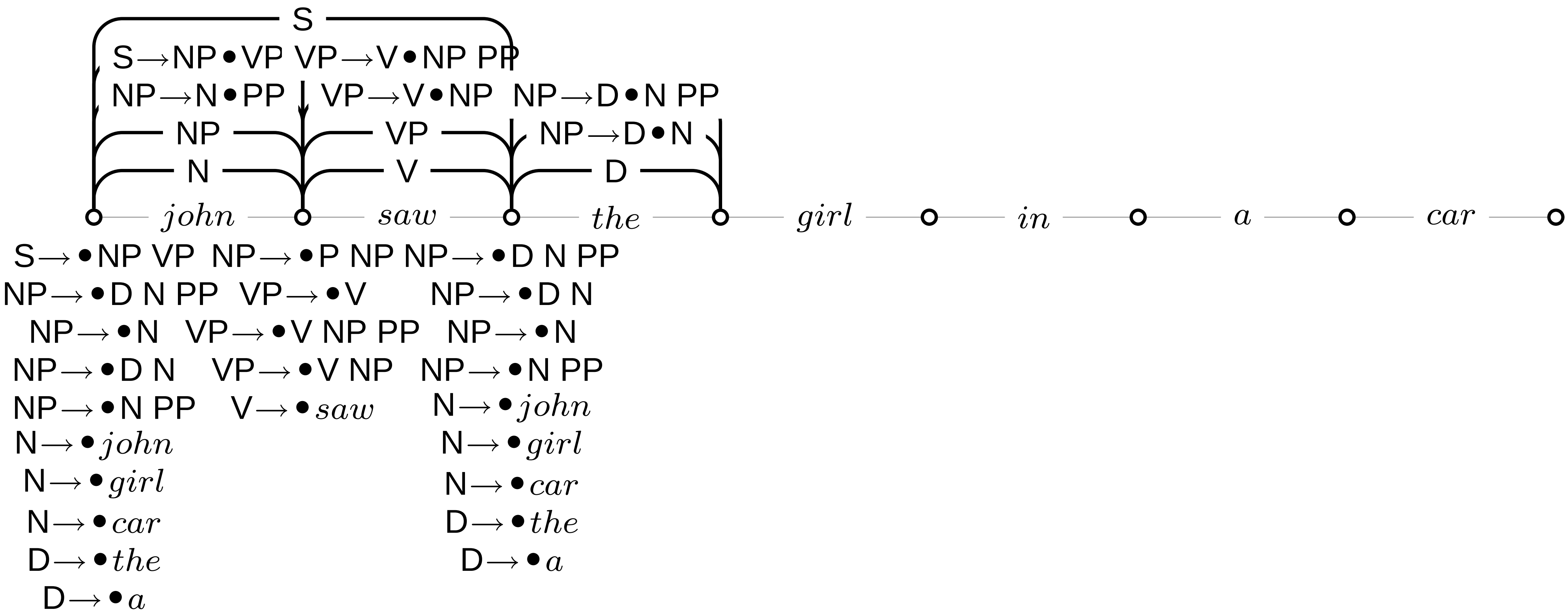

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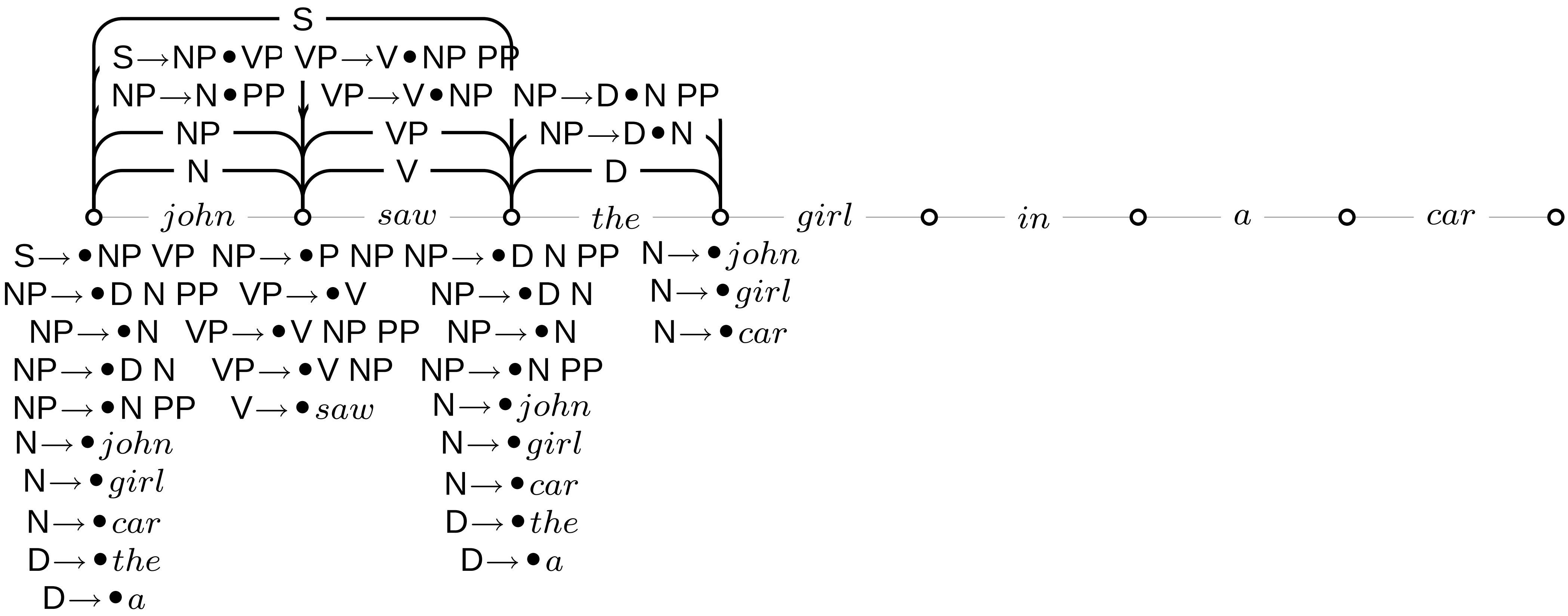

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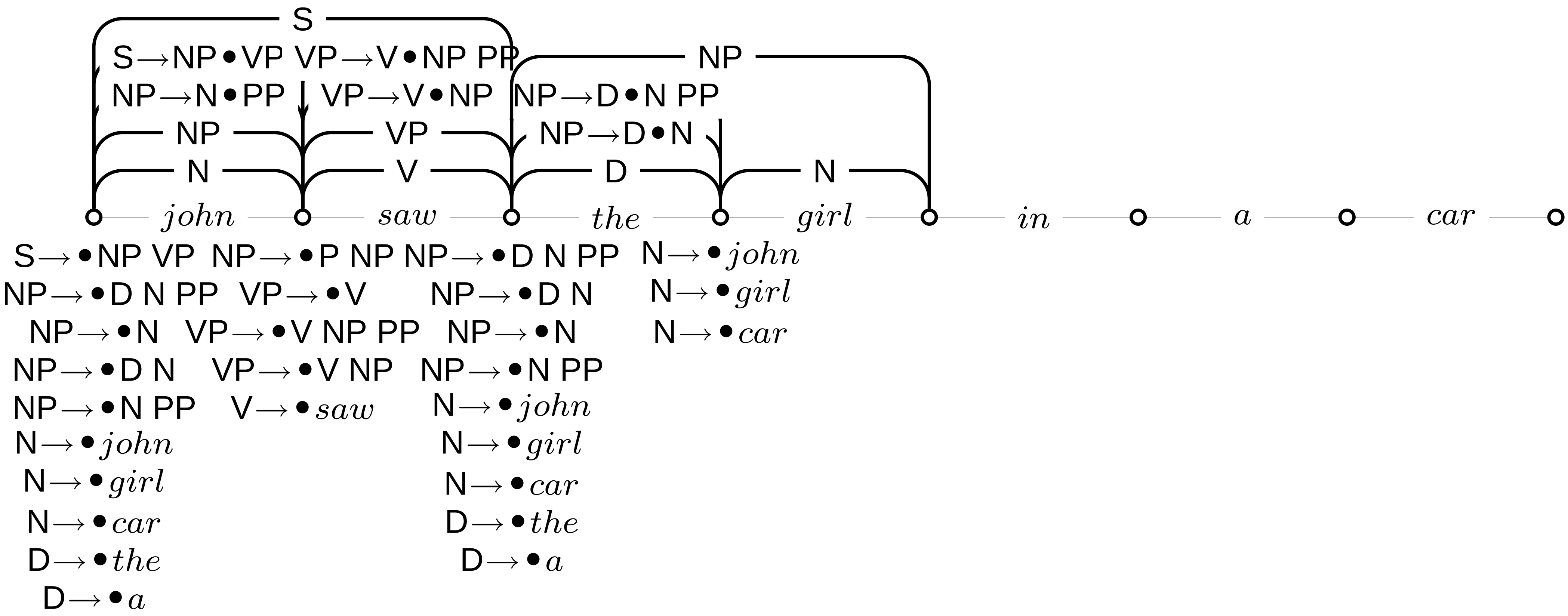

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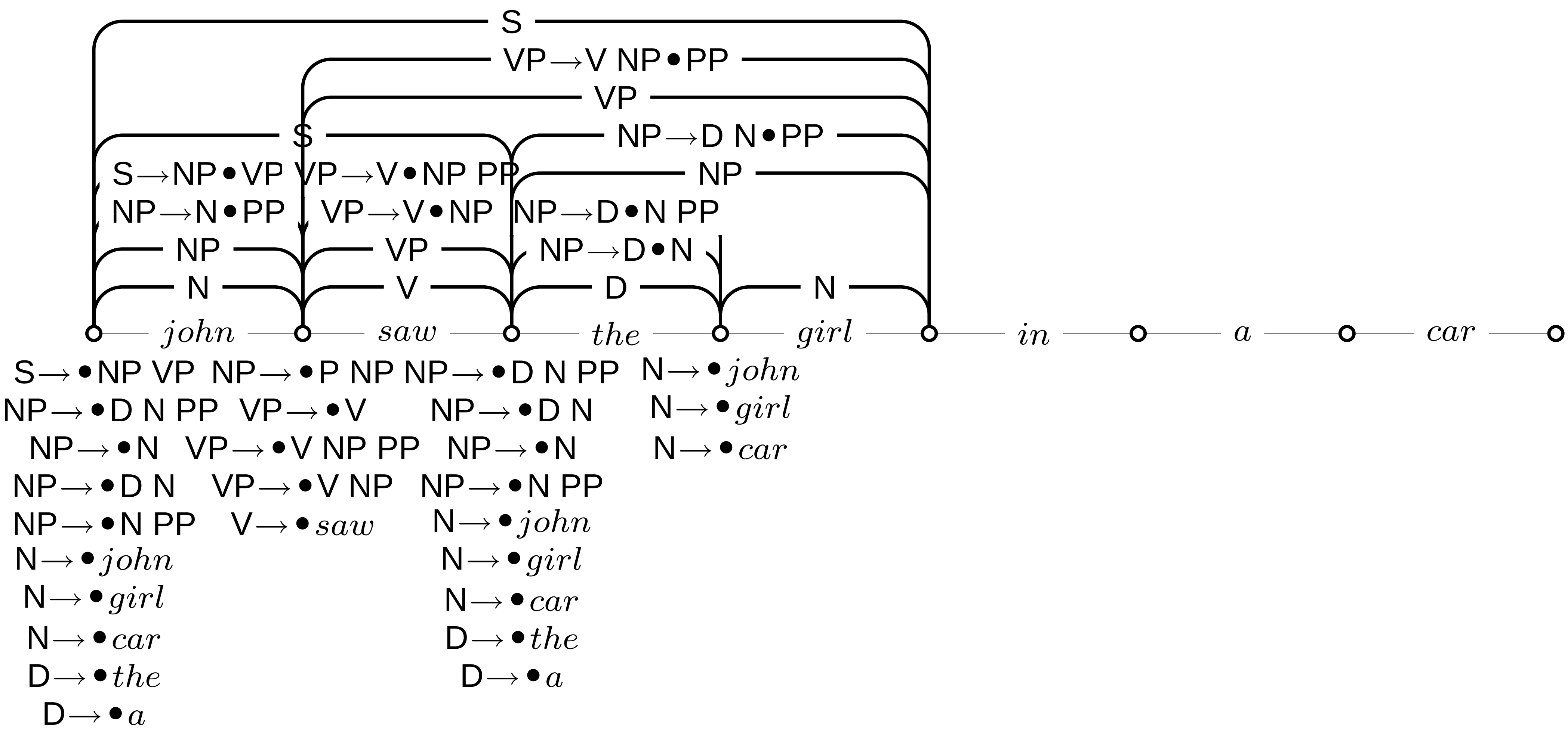

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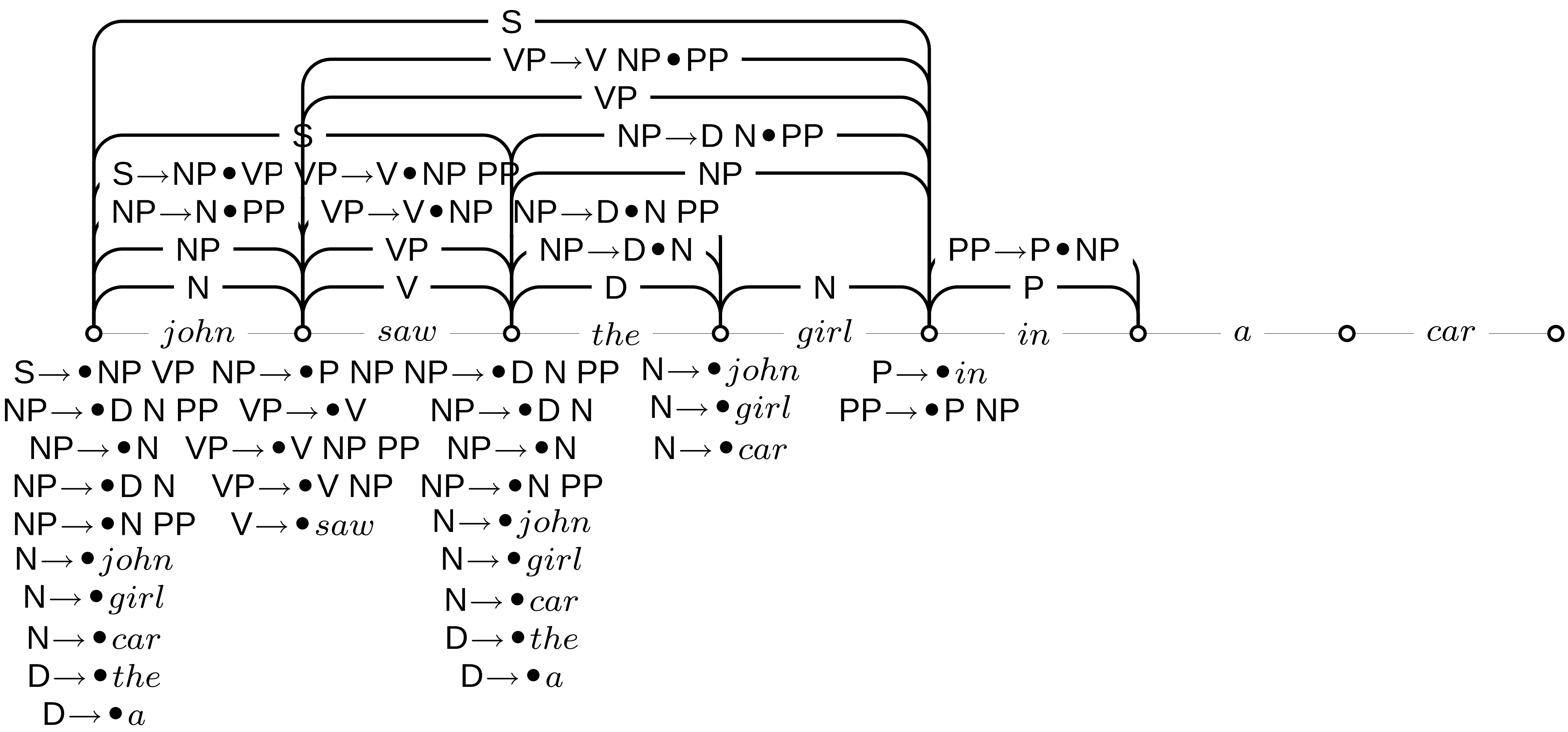
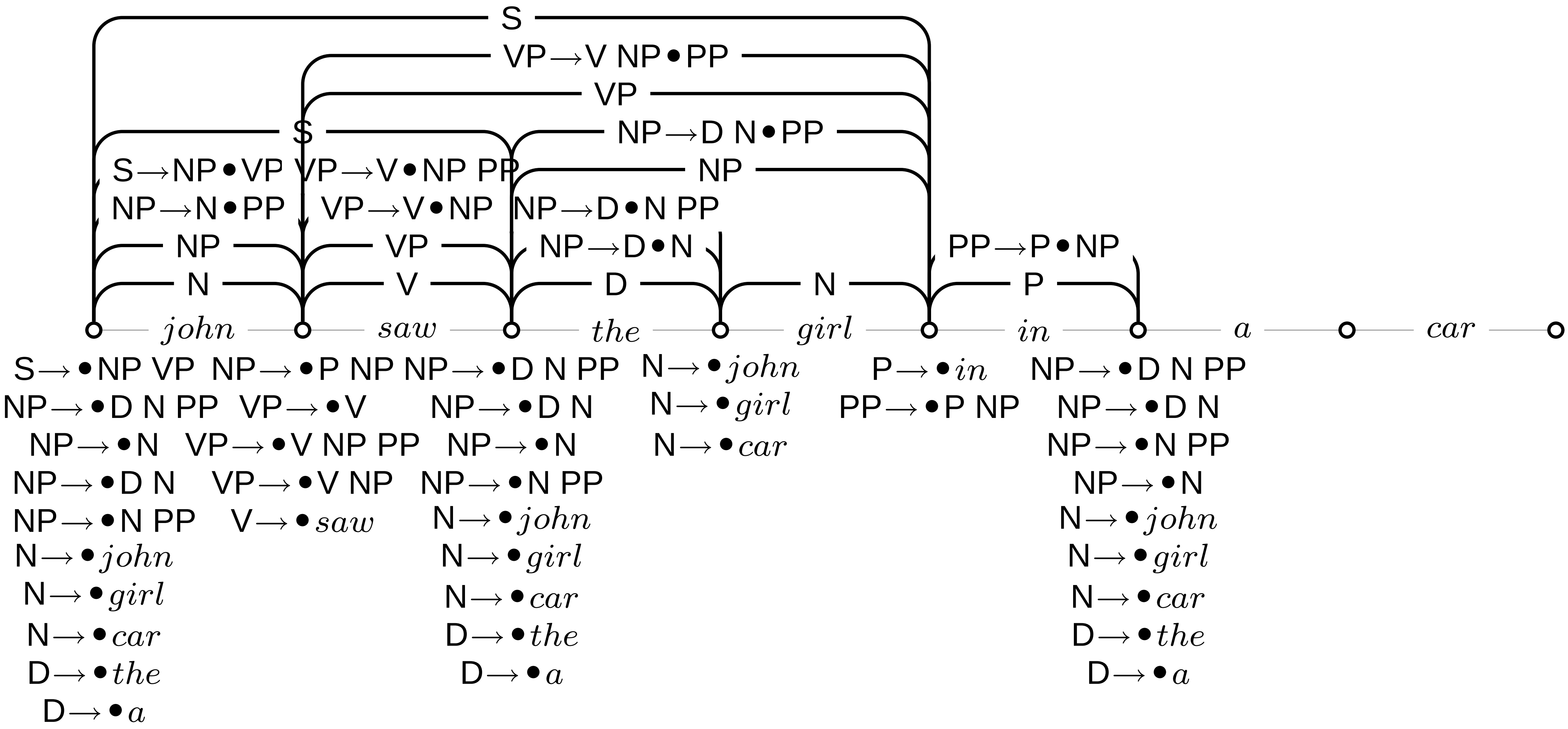
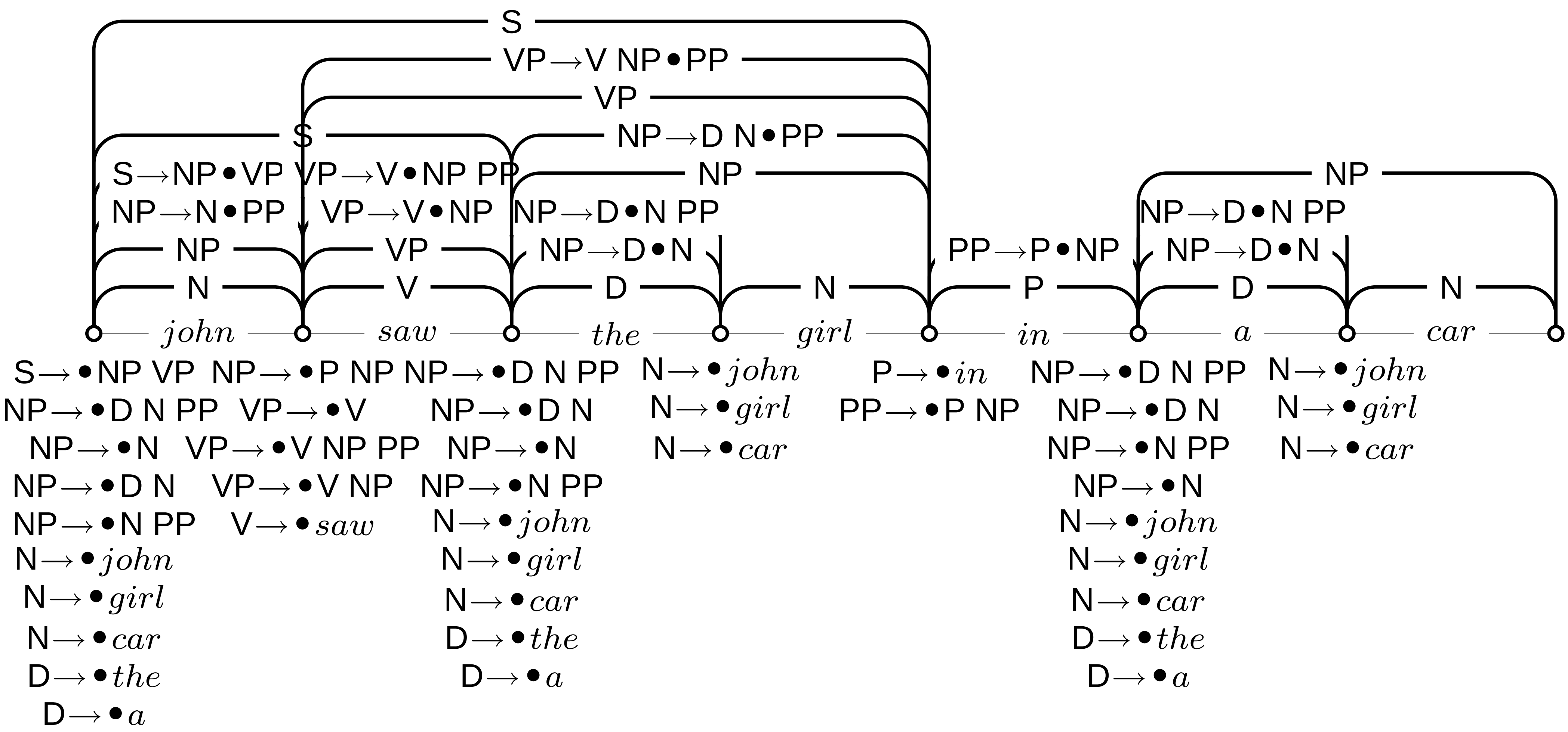


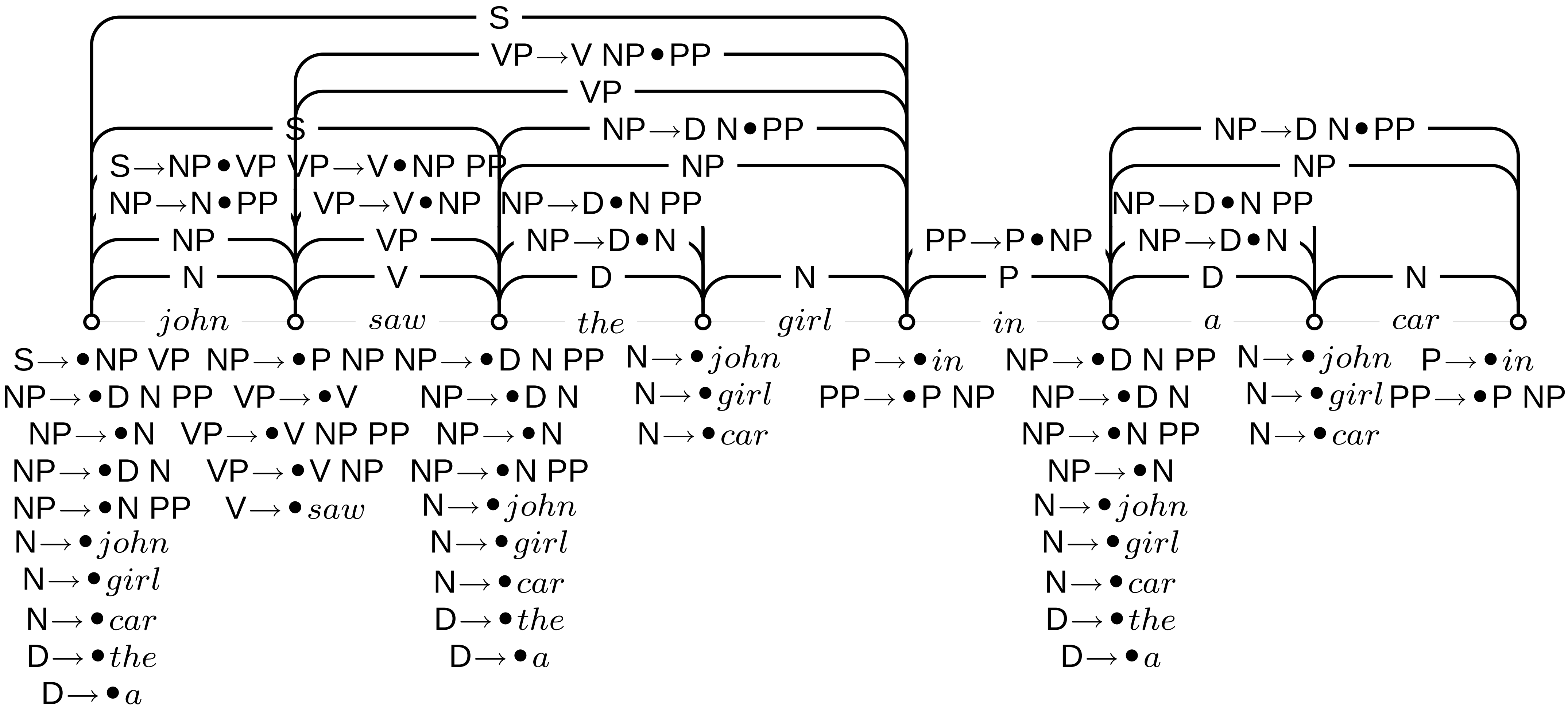
Earley Parsing Example



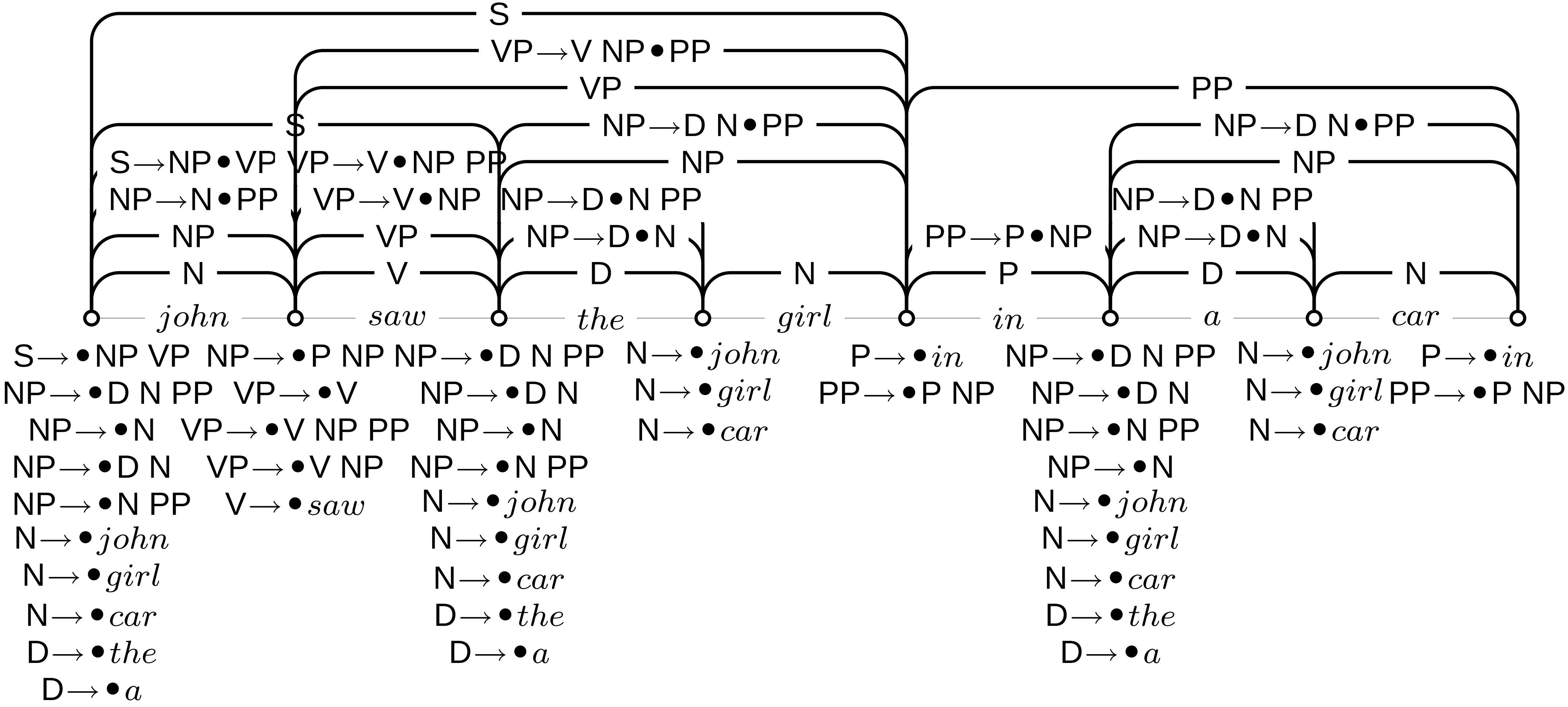
Earley Parsing Example



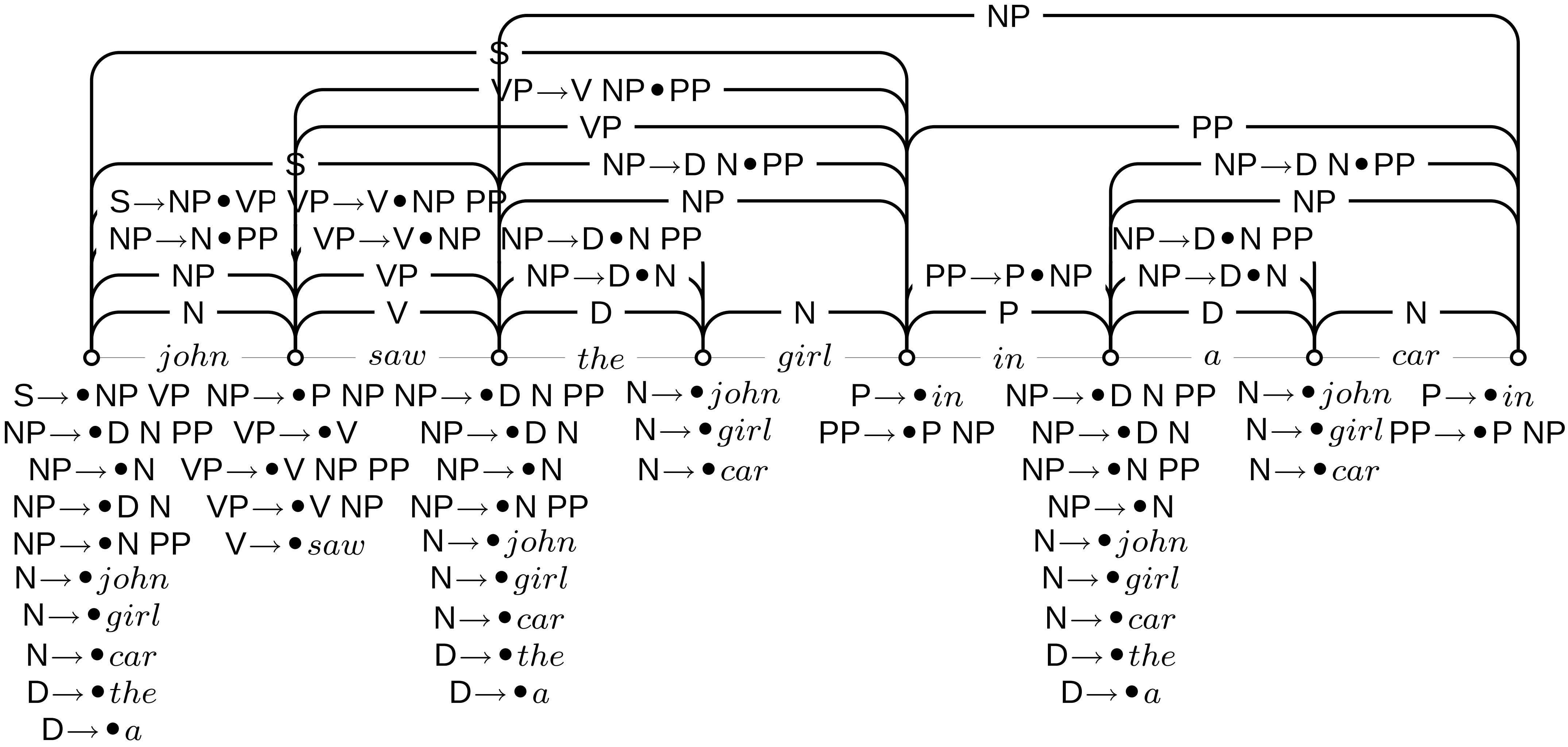
Earley Parsing Example



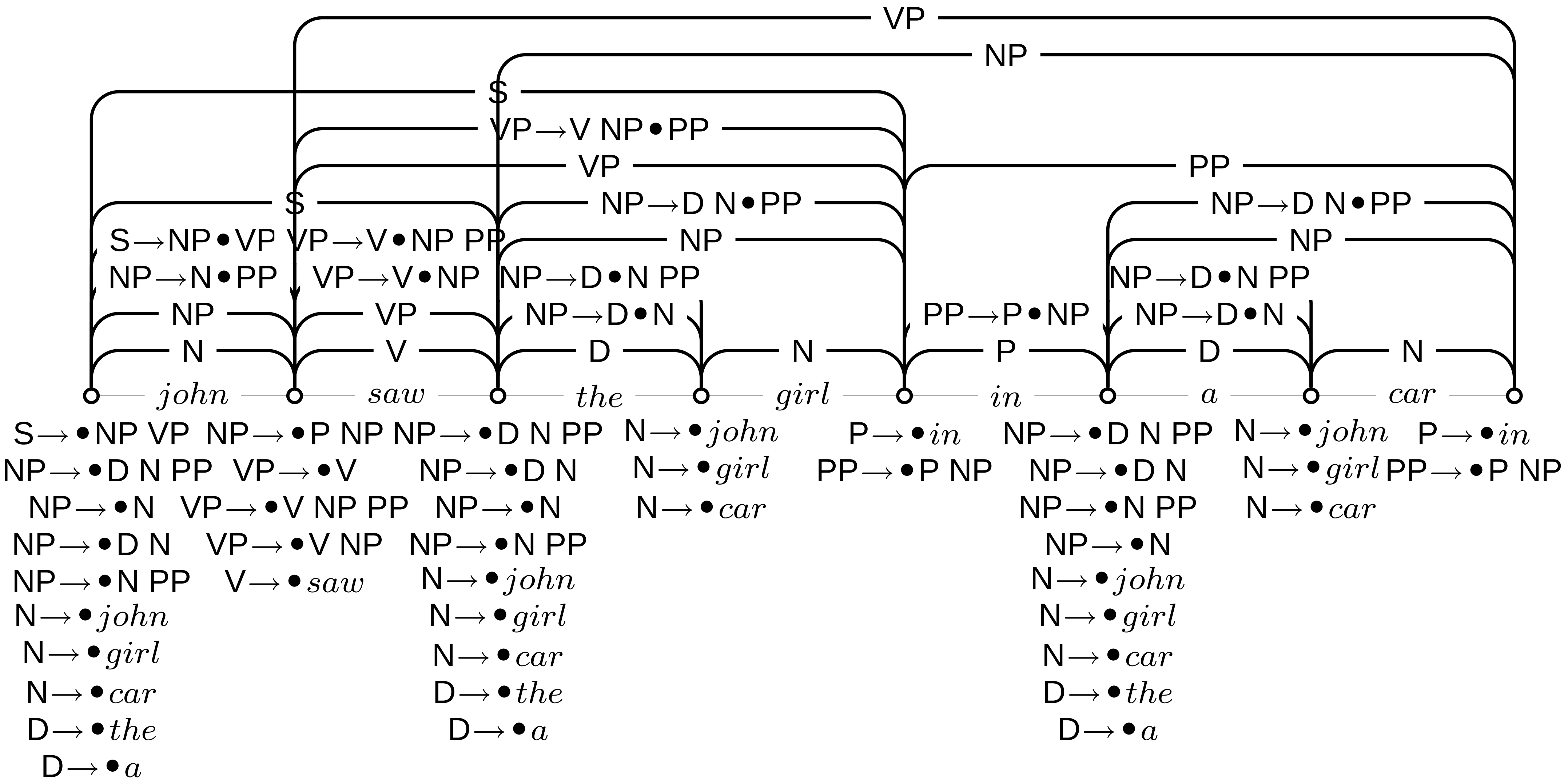
Earley Parsing Example



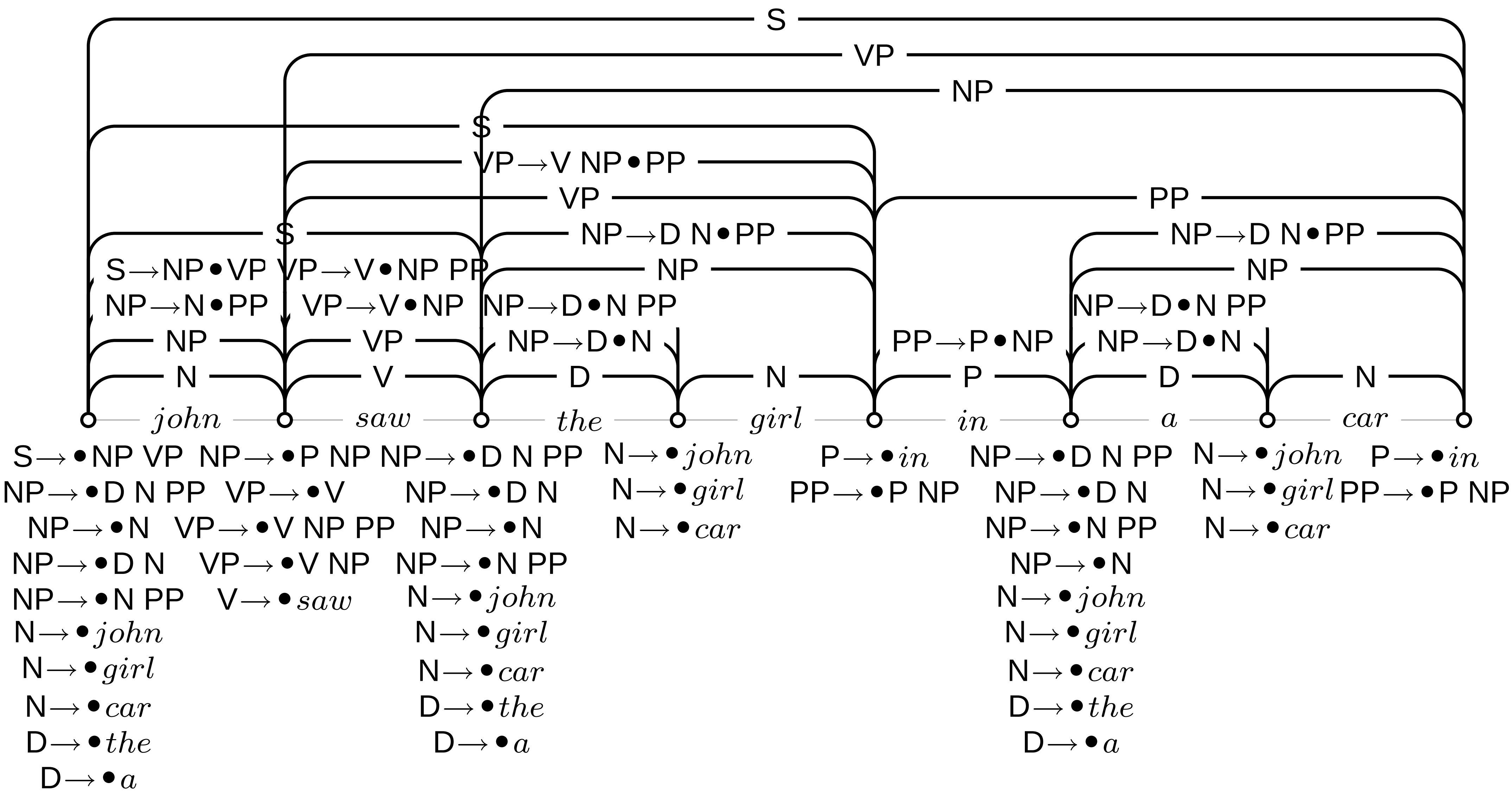
Earley Parsing Example



Earley Parsing Example



Earley Parsing Example



Earley Parsing – Summary

- The number of useless items is reduced
- Superior runtime for unambiguous grammars: $\mathcal{O}(n^2)$
- Valid prefix property
- Not all sub-derivations are computed

Left Corner Parsing

- Observation: Earley parsing predicts items that can not succeed
- Idea: predict only items that can also be derived from the leftmost terminal item
- Formalization: left-corner relation
  - ➤ $A >_l B \Longleftrightarrow \exists \beta : A \to B\beta \in P, B \in \Sigma \cup N$
  - ➤ $A >_l^*$ is the transitive closure of $>_l$
- Reformulation of the prediction step:
  - ➤ If $(A \to \beta \bullet Y\alpha, i, j)$ and $(B, j, k) \in \mathcal{C}$, with $B \in \Sigma \cup N$ add $(C \to B \bullet \gamma, j, k)$ if $Y >_l^* C$
- This formulation also avoids the zero-length predictions with the dot in initial position

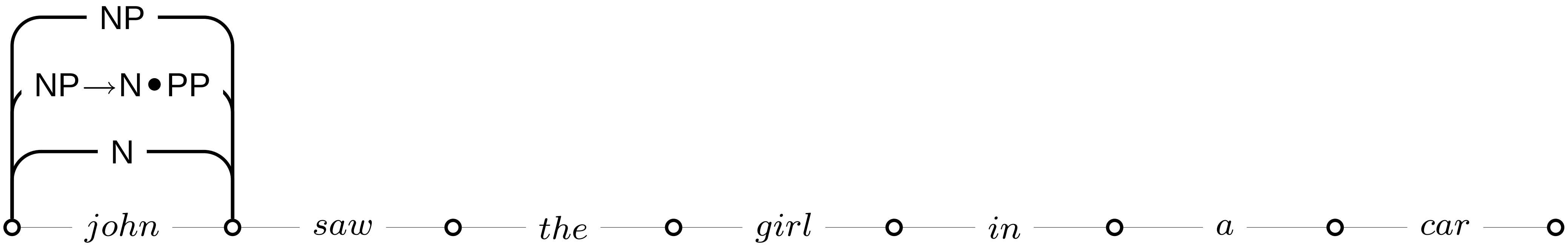
Left-Corner Example

$\circ$ — *john* — $\circ$ — *saw* — $\circ$ — *the* — $\circ$ — *girl* — $\circ$ — *in* — $\circ$ — *a* — $\circ$ — *car* — $\circ$

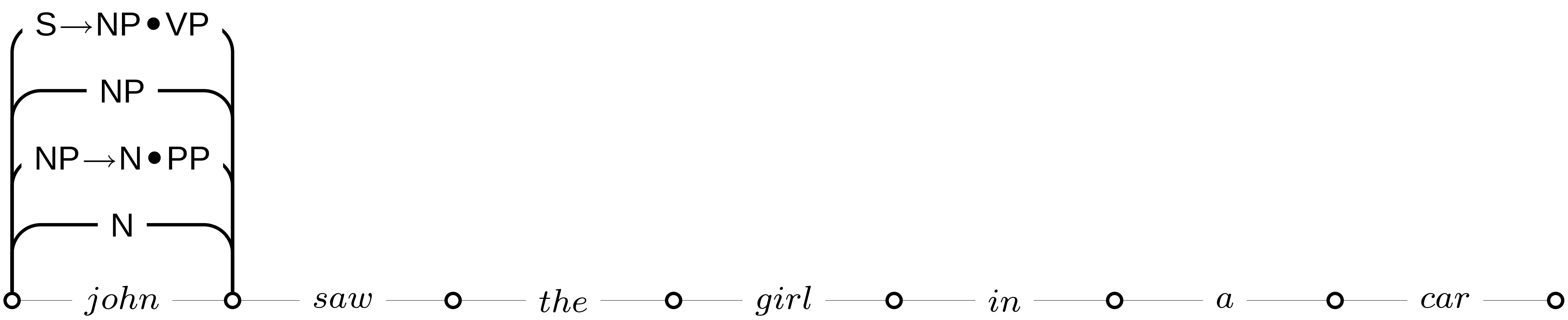
Left-Corner Example

Left-Corner Example

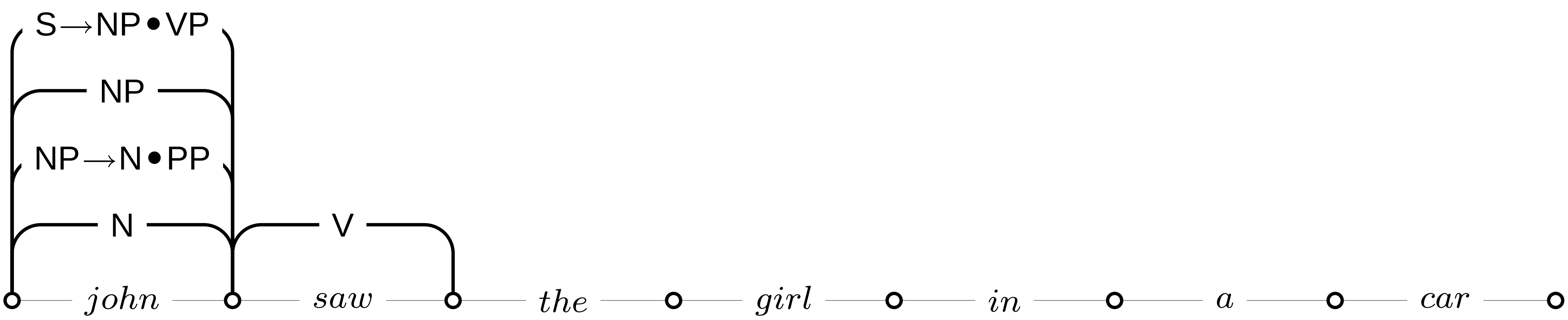
Left-Corner Example

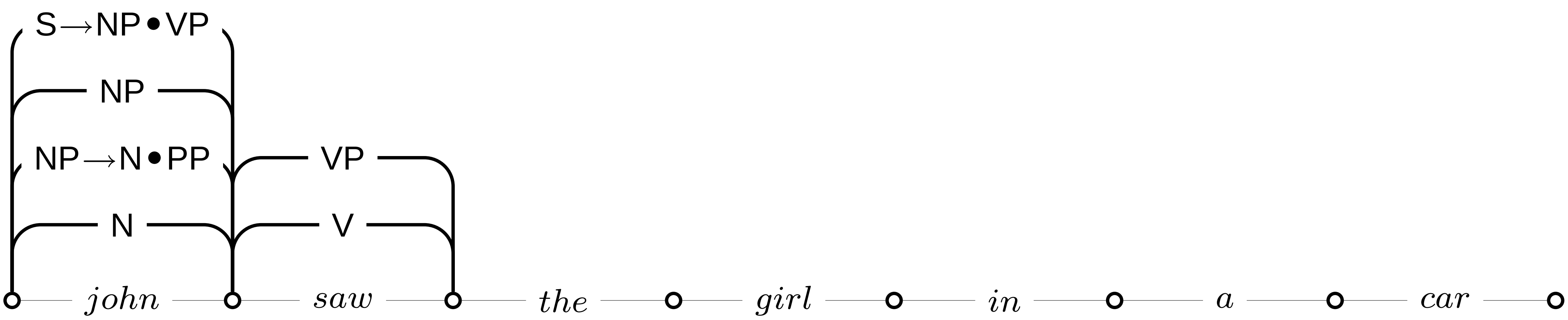


Left-Corner Example



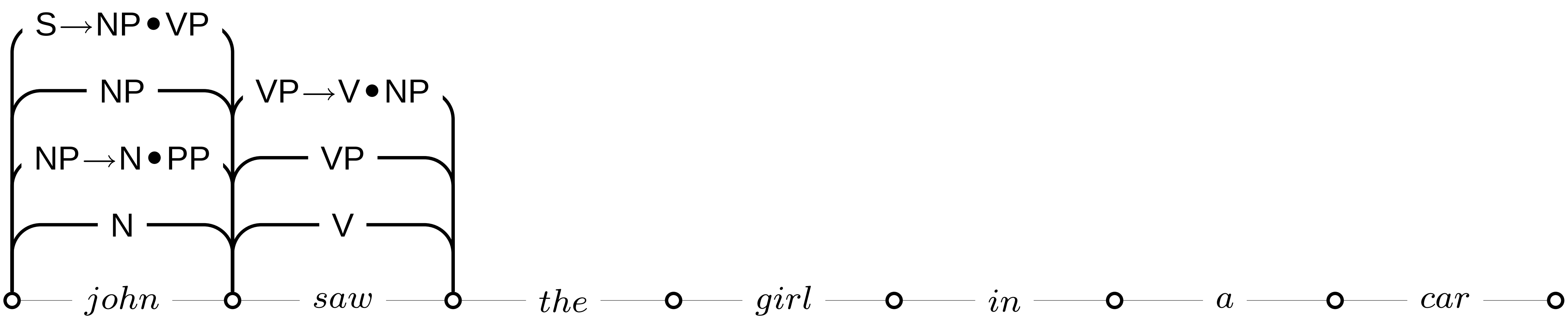
Left-Corner Example



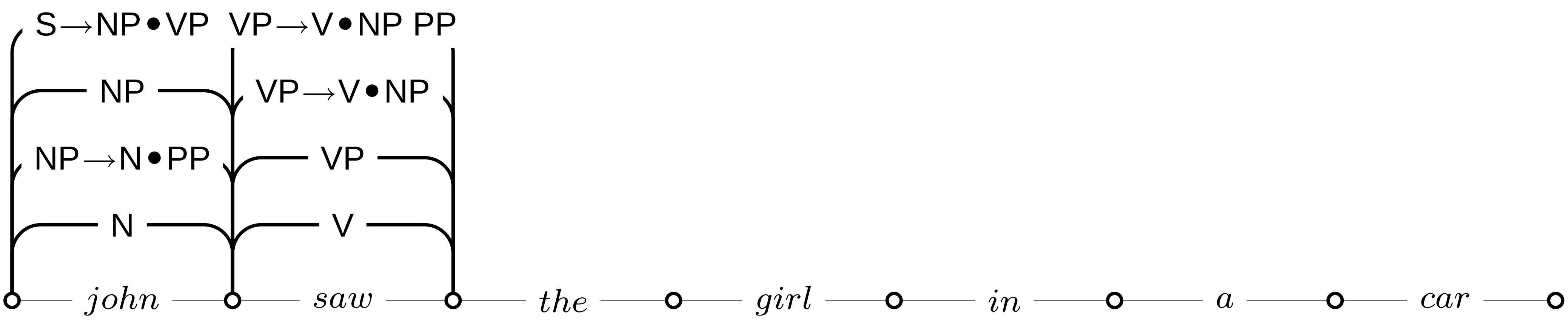


S→NP•VP
NP
NP→N•PP
VP
N
V
john
saw
the
girl
in
a
car

Left-Corner Example



Left-Corner Example



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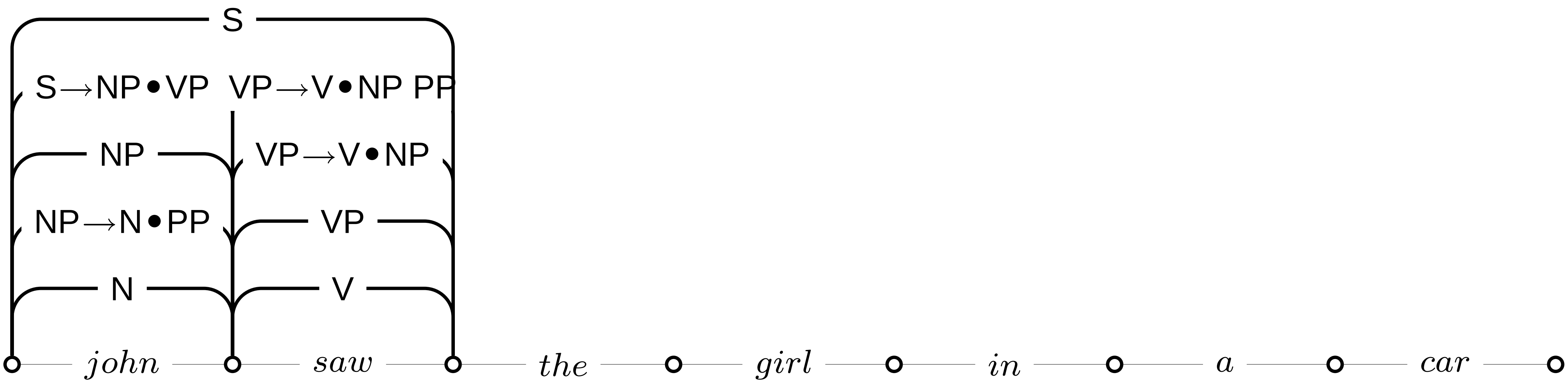

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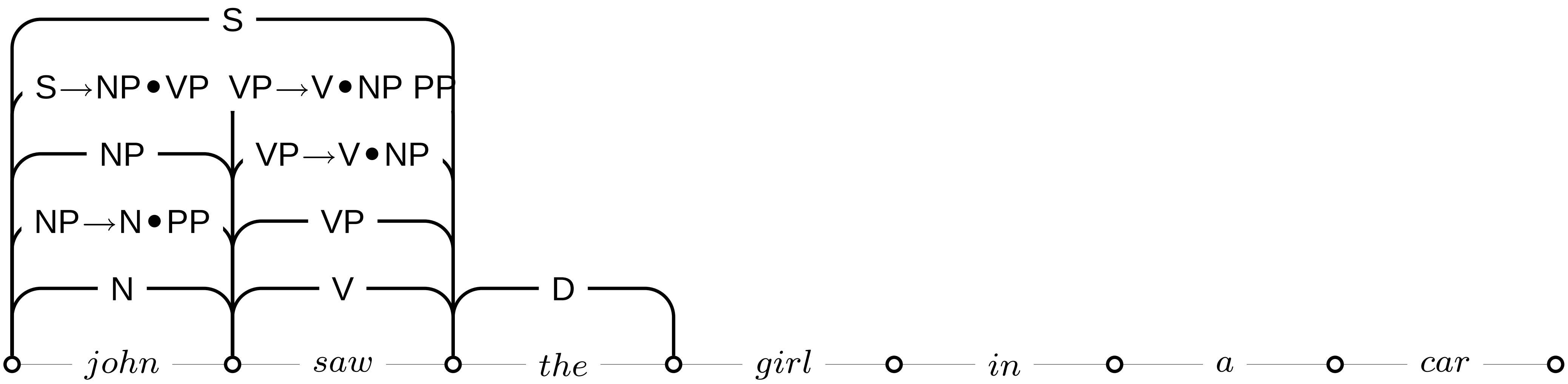
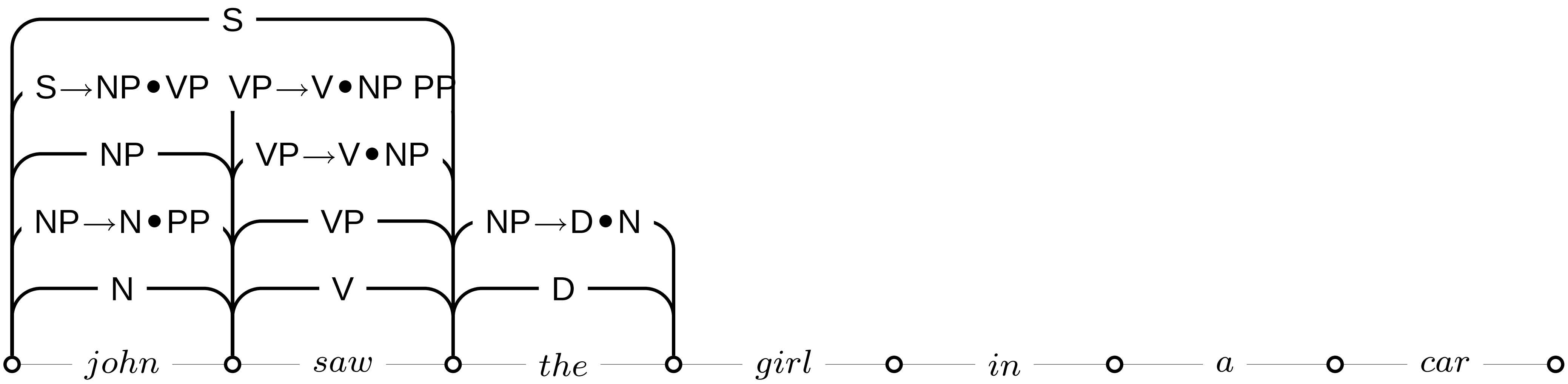
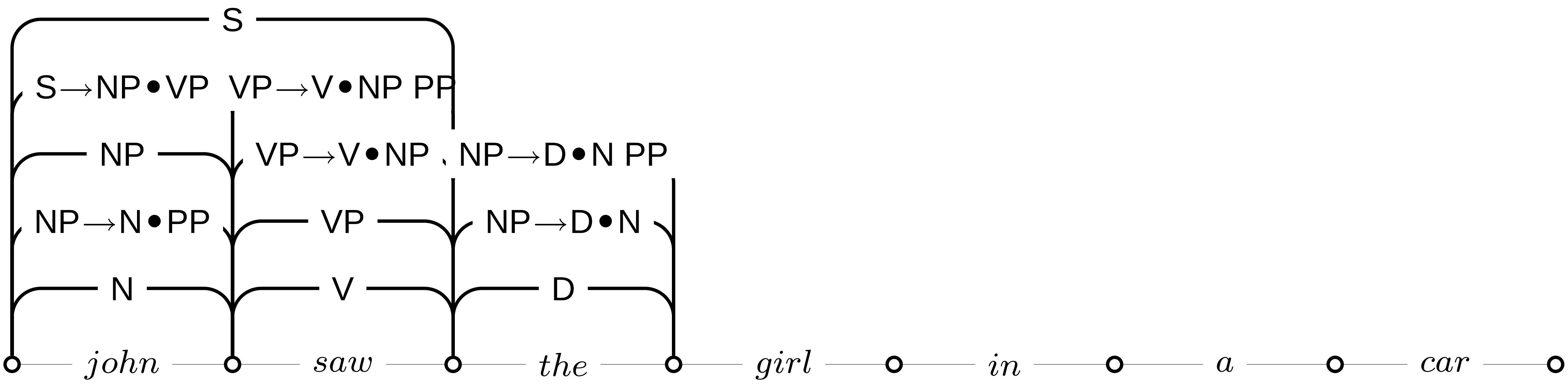


Left-Corner Example



Left-Corner Example



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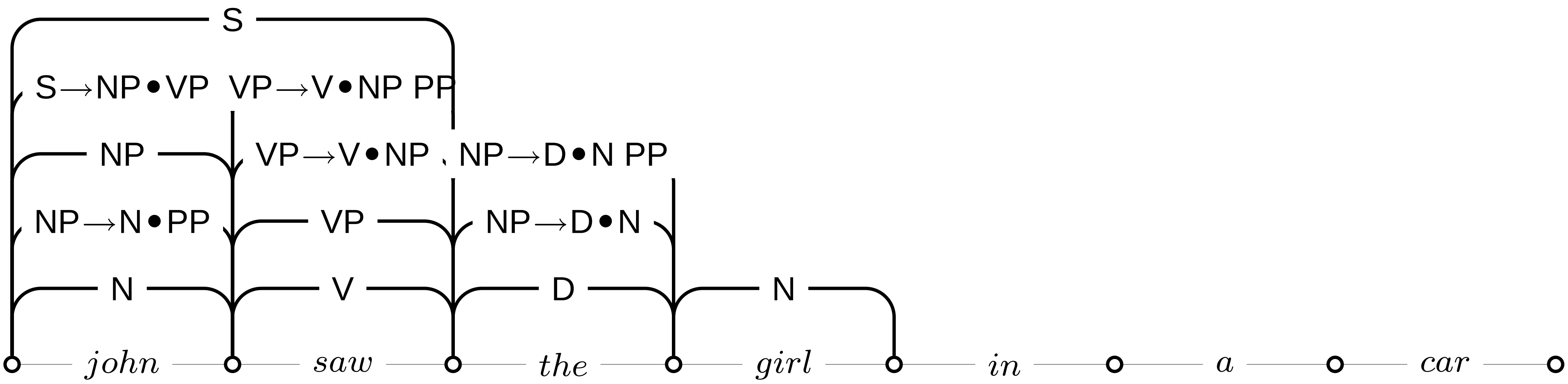

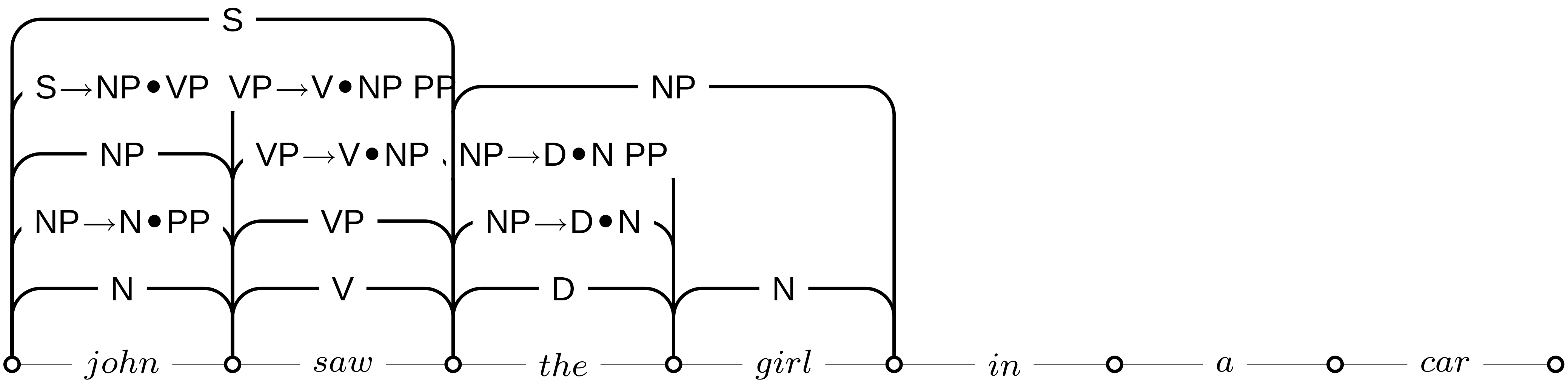
S
S→NP•VP
VP→V•NP PP
NP
NP
VP→V•NP
NP→D•N PP
NP→N•PP
VP
NP→D•N
N
V
D
N
john
saw
the
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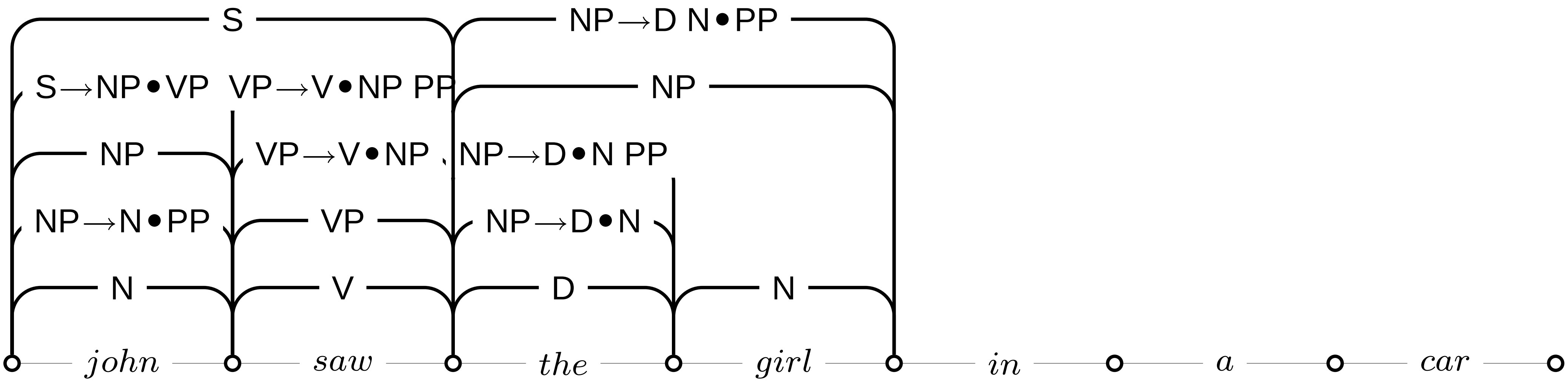
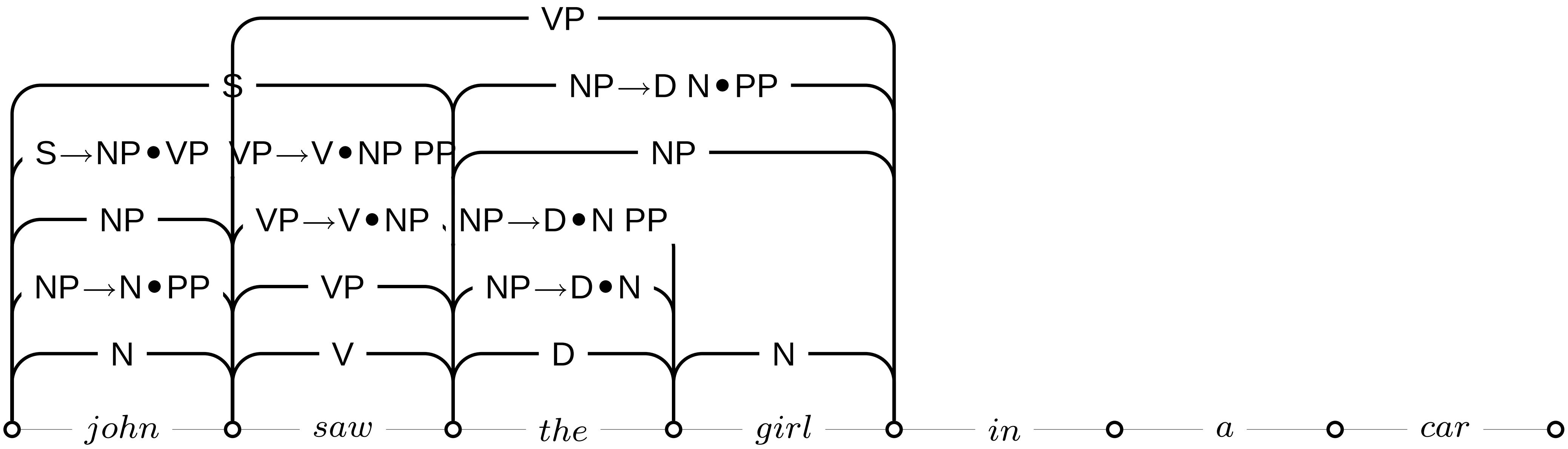
Left-Corner Example



Left-Corner Example



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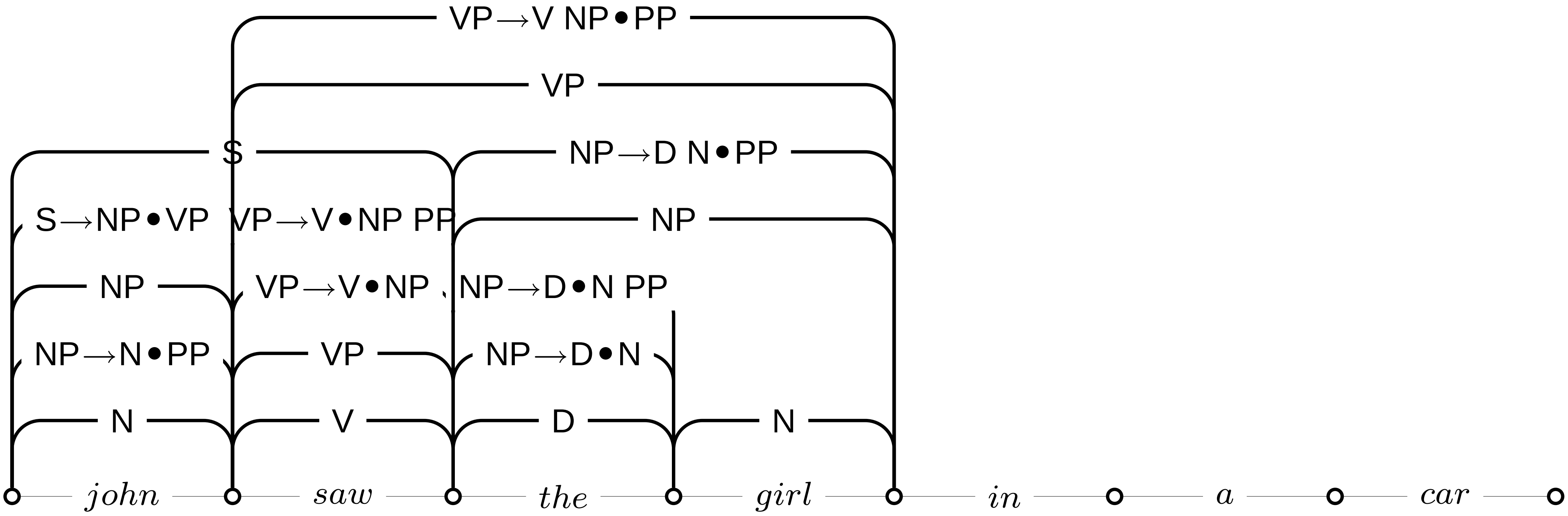
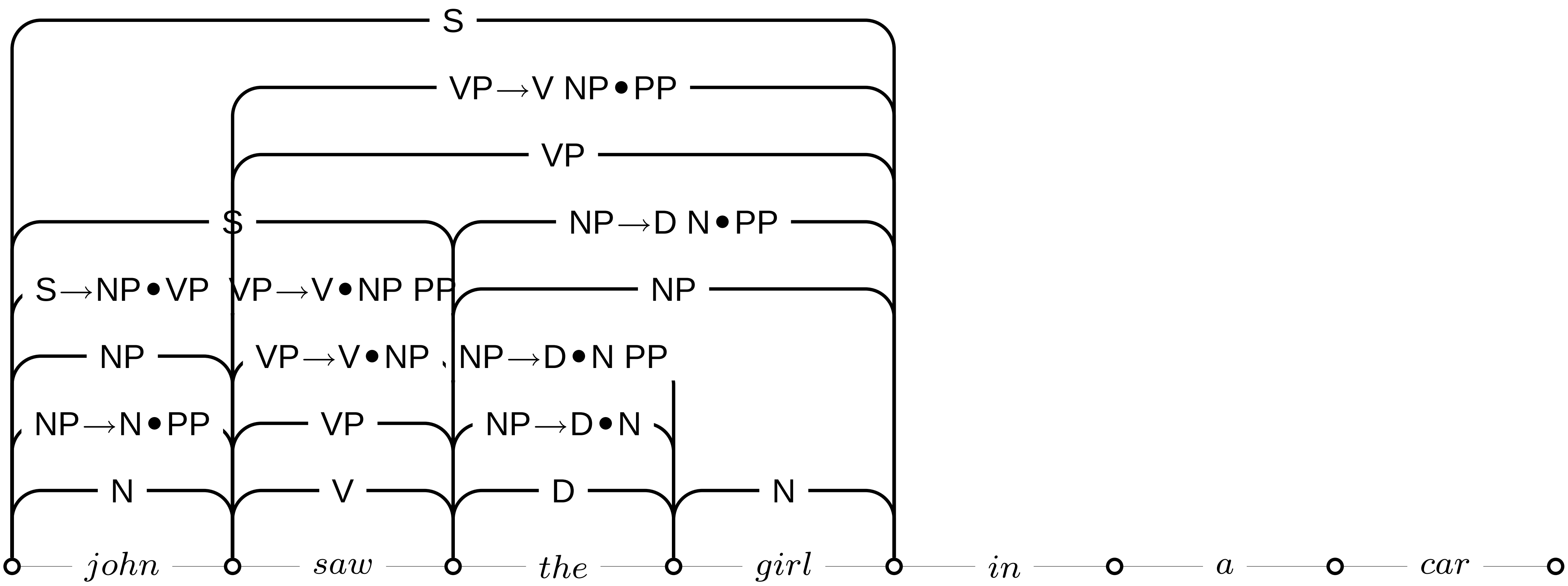


Left-Corner Example



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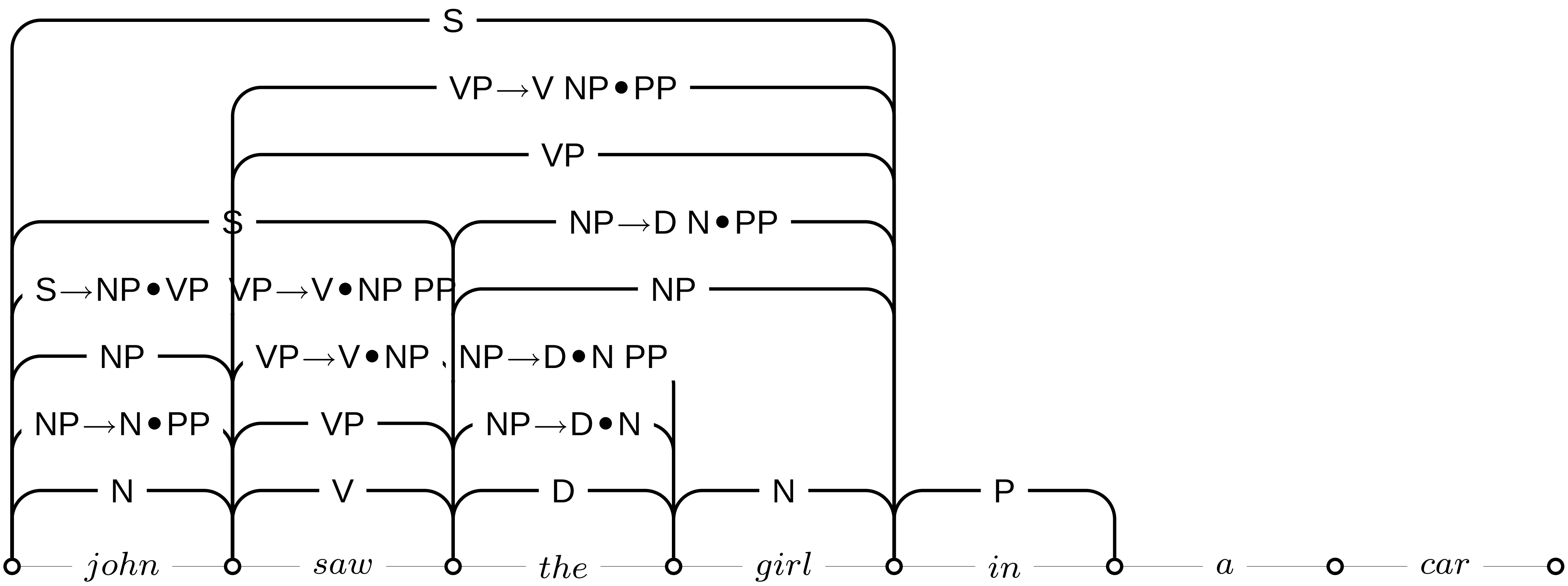

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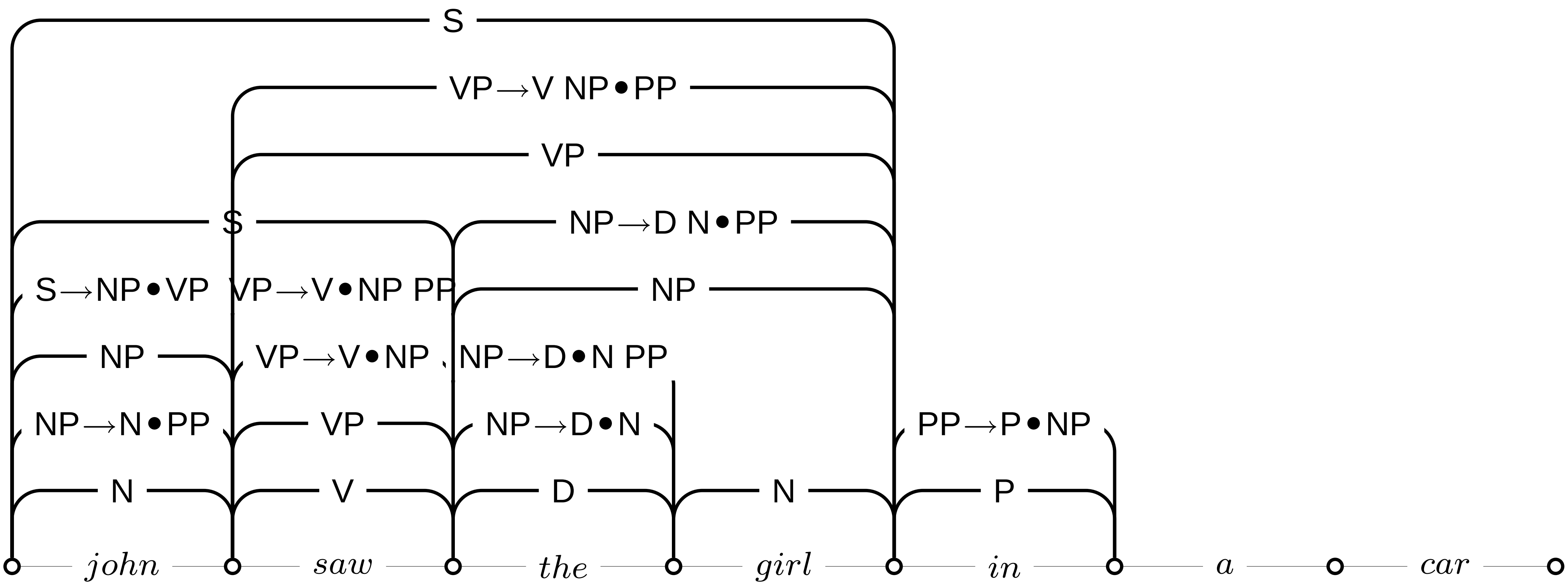

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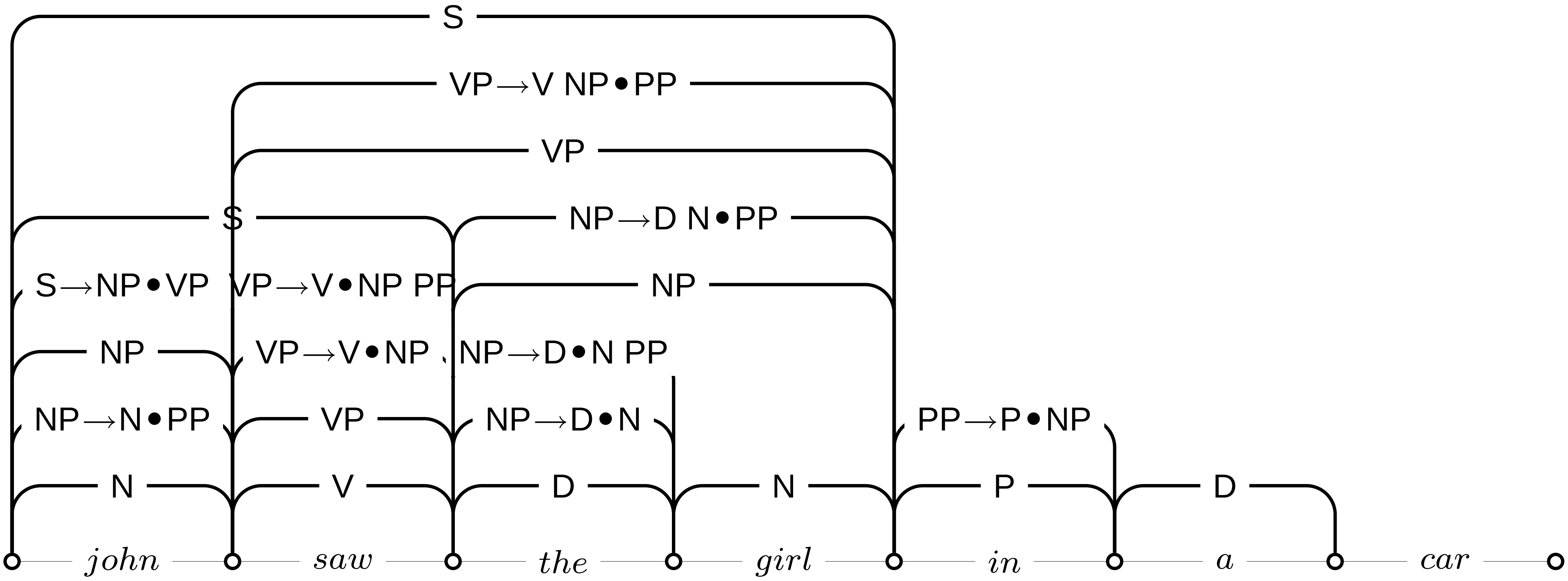

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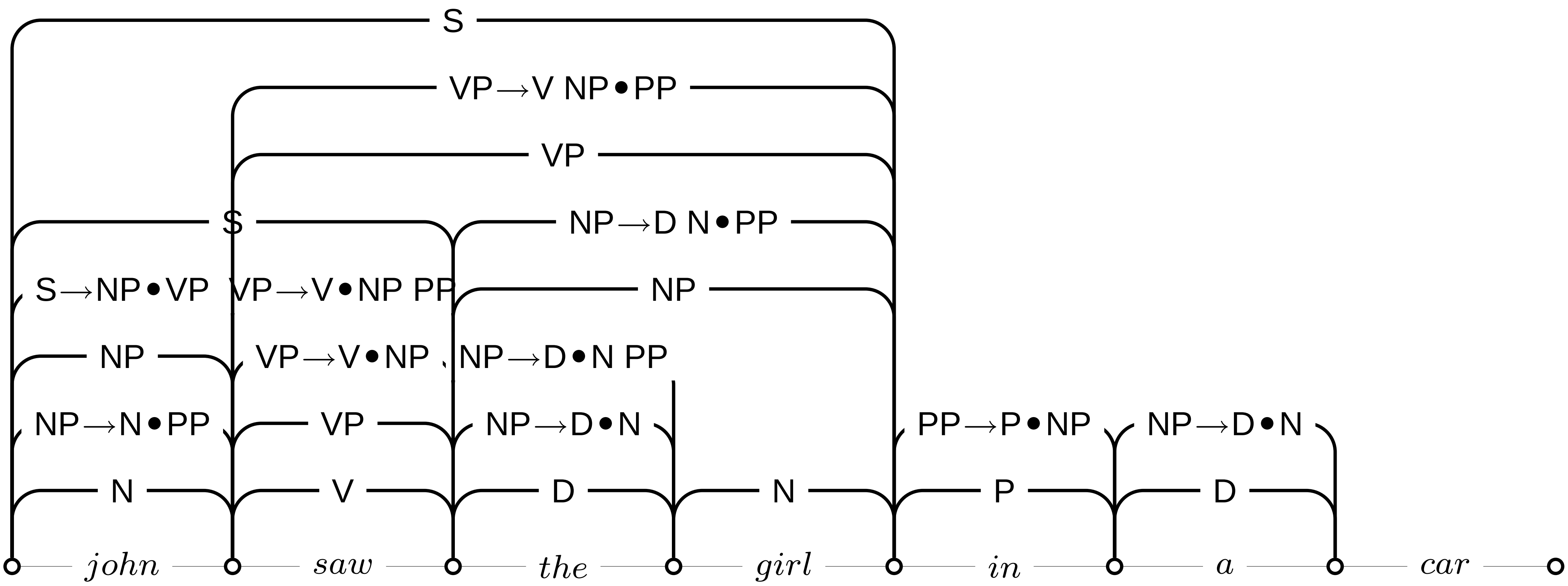

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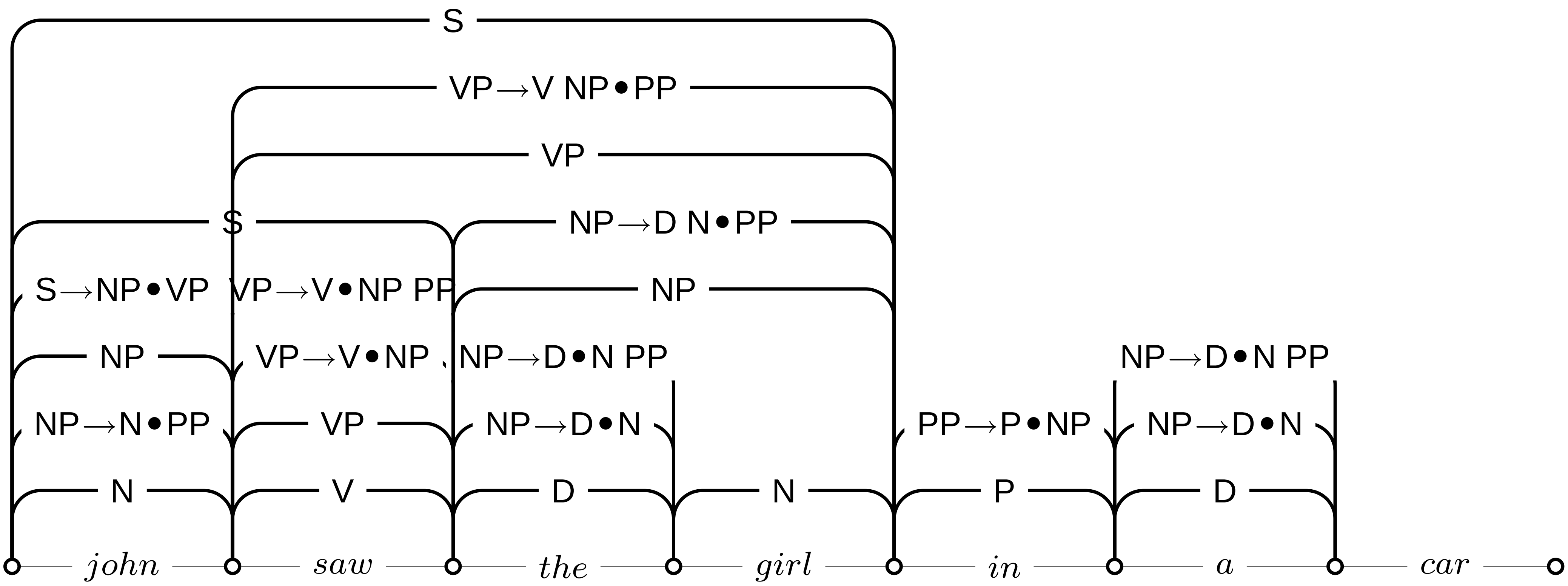

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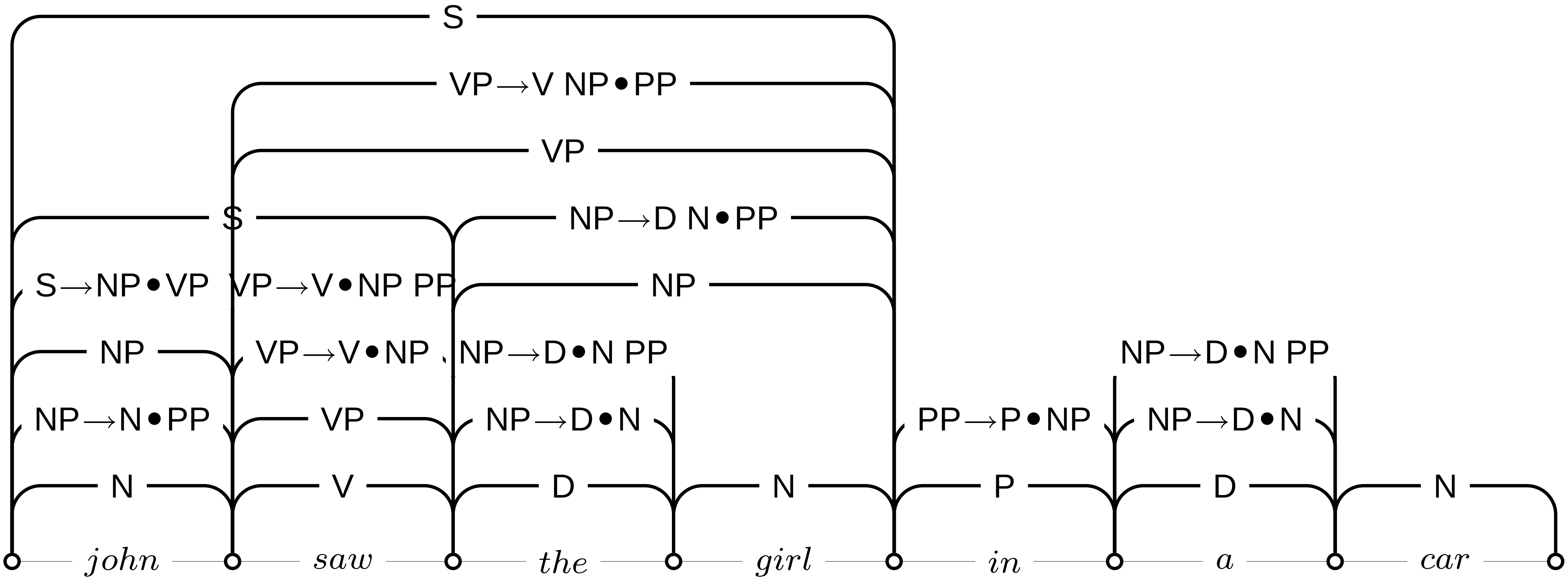

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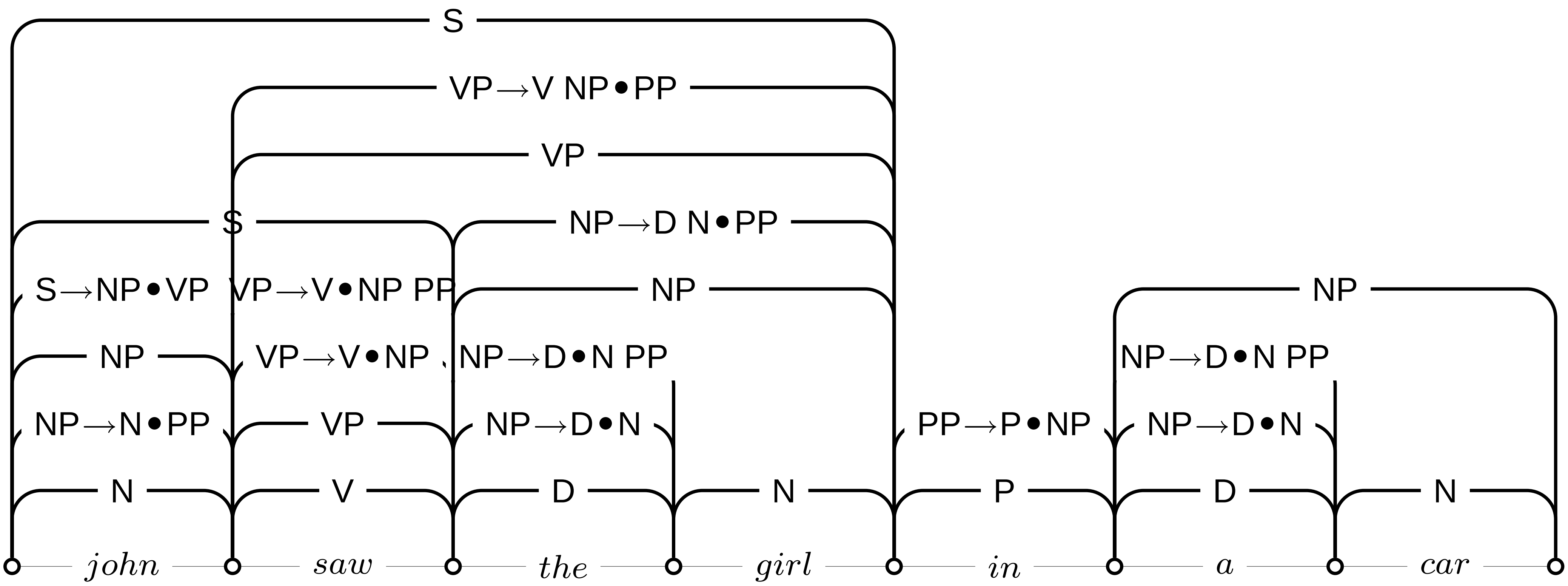

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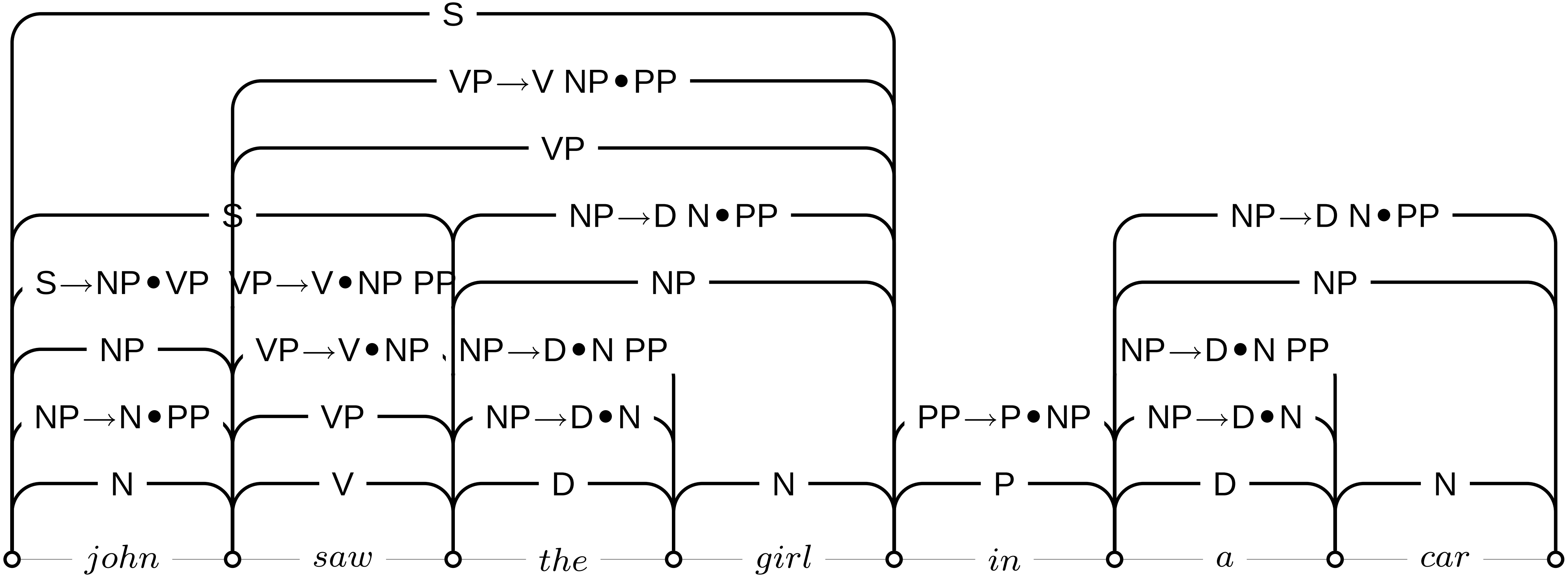

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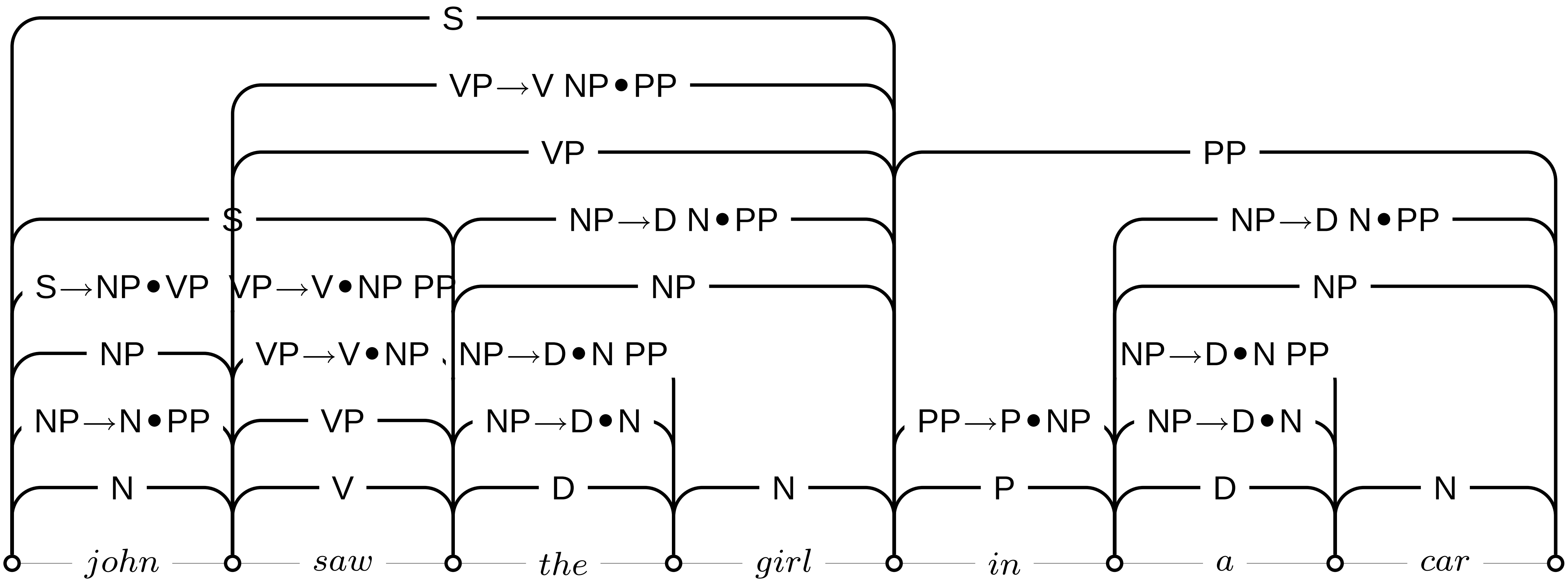

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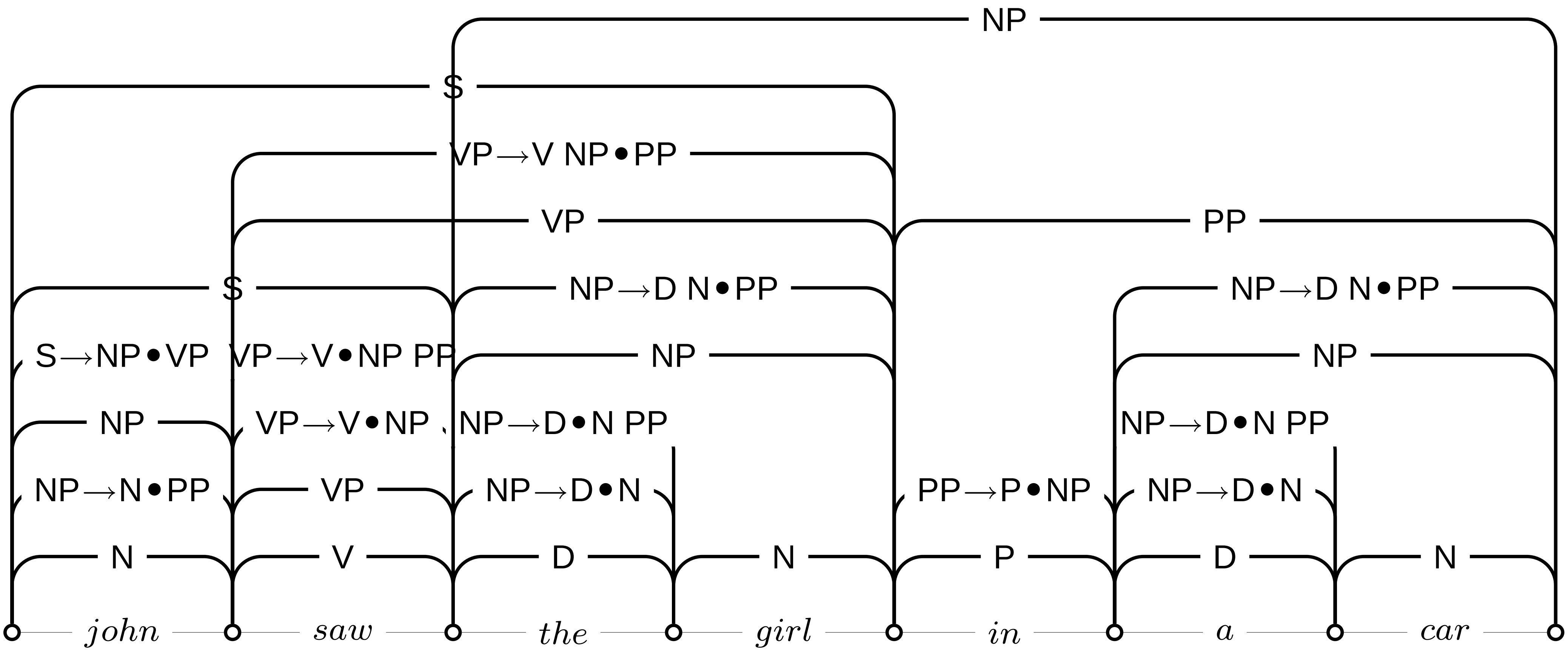

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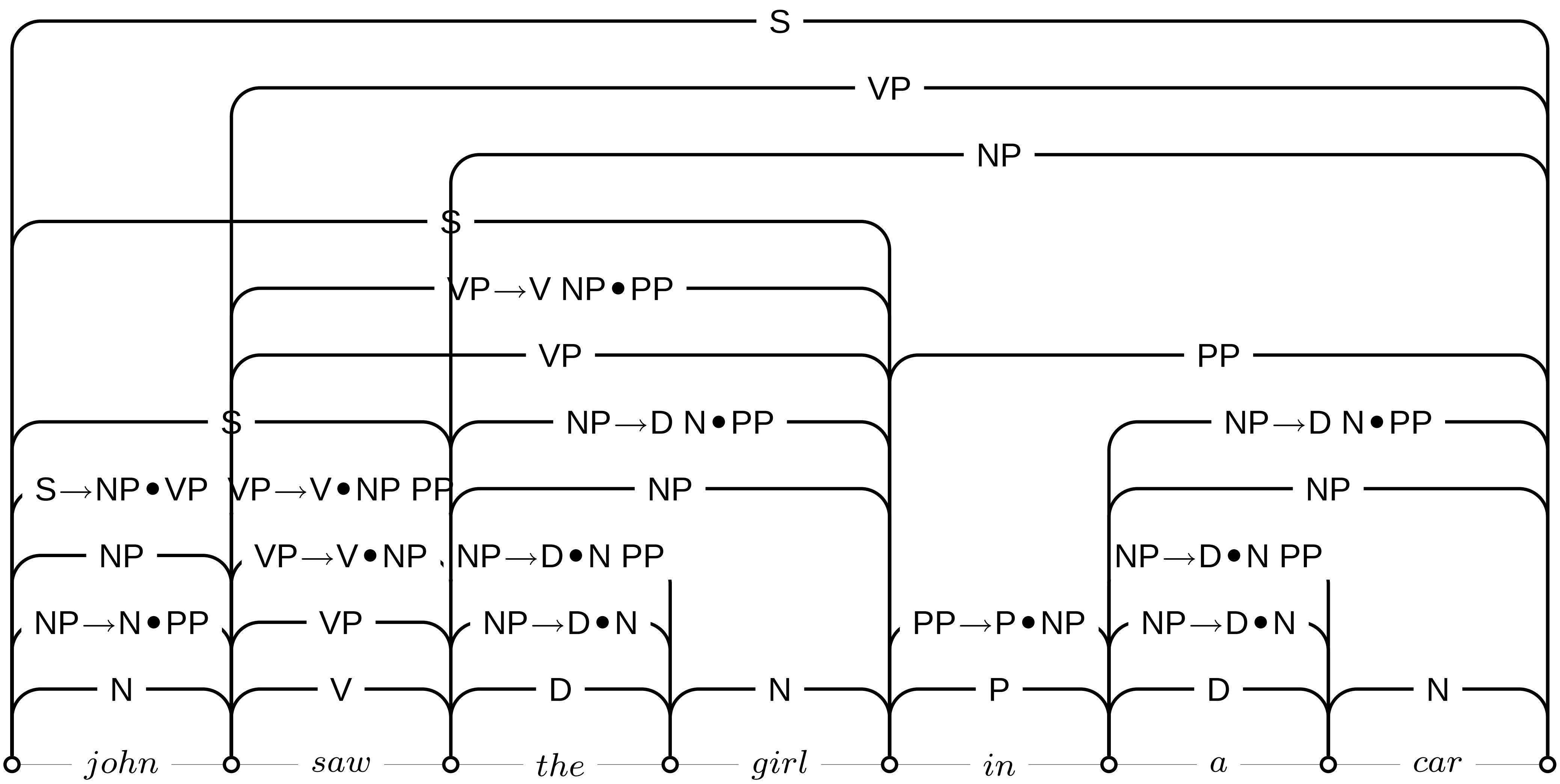

Context-Free Parsing: Summary

- Context-free grammars provide you with a finite set of infinitely embeddable brackets
- Two main approaches to CF recognition: top down (goal-driven) and bottom-up (data driven)
- Storing sub-derivations for re-use (*dynamic programming*) in a chart lead to a polynomial algorithm with worst case n^3
- The chart offers a compact (polynomial size) storage for a possibly exponential number of results
- Earley and Left Corner Parsing improve the average runtime over the naïve CYK algorithm, and have a better worst case complexity for some classes of context-free grammars